

Modal Analysis, Health Monitoring, and Train-Induced Resonance of Multi-Span Bridges

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Abstract: Resonances of railway bridges due to the passage of trains have been mainly investigated for single-span bridges. When multi-span bridges are considered, it is of interest whether stronger resonance amplifications compared to a single-span bridge must be taken into account. Measurements of eight multi-span structures are presented with their natural frequencies and mode shapes: wooden floor beams, a concrete beam, a steel truss footbridge, three road bridges, and two railway bridges. Typical effects are shown, such as multiple fundamental frequencies due to parallel beams and multiple spans, close eigenfrequencies of different mode shapes, clusters for simply supported or weakly coupled spans, and regularly separated frequencies for continuous beams and bridges. The mode shapes are generally global, but sometimes more local mode shapes exist; for example, a longer midspan is dominant in the first eigenmode. The consequences of multi-span bridges for train passages are discussed based on a frequency-domain theory in which three spectra determine the resonance excitation: the axle-sequence spectrum of the train, the transfer function of the bridge, and the modal force spectrum. The method is applied to the two railway bridge examples, which are treated as single-span bridges at first, and then these examples are extended to two-, three-, and multi-span bridges. The resonance of a railway bridge occurs when a maximum of the axle-sequence spectrum (the second or third car-length frequency in the examples) coincides with an eigenfrequency of the bridge. The resonance amplitude is determined by the mode shape, which results directly in the modal force spectrum and the modal mass, for which some rules are presented, as well as for the resonance amplitude of multi-span bridges.

Author keywords: Railway bridge; train passage; resonance; multiple spans; mode-shape spectrum

Introduction

When trains pass over bridges, the response to a moving load is higher than that for a static load. Higher amplifications occur due to the repeated axle loads. For special combinations of train speed, train configuration, bridge length, and eigenfrequency, a strong resonance can occur. For high-speed trains on short bridges, the amplitudes were sometimes so high that the ballasted track was damaged.

The topic of resonances of railway bridges has attracted many researchers in the past,¹⁻⁴ and many articles about methods, phenomena, and standards have been published.³⁻⁷ The resonance of bridges under train passage is a complex phenomenon in which, besides amplification, cancellation (of resonances) is also observed.⁸⁻¹⁰ Most theoretical analyses are performed by time-domain methods, and only a few articles use frequency-domain methods.^{4,11,12} A modal analysis of the bridge is usually a fundamental step in the

investigation. In the attempt to get simpler methods and rules, the problem is divided into the phase of bridge passage and the free vibrations after the bridge passage, e.g.,¹² and into the passage of the static train loads and the full vehicle-bridge interaction, e.g.^{13,14} Finally, many experiments on bridge resonances under train passages have been reported, for example, in.¹⁵⁻¹⁹ All these articles deal with single-span bridges, in many cases with simply supported bridges. Only a few articles present examples or methods for multi-span railway bridges.²⁰⁻²⁵ More general rules for multi-span bridges are rarely found.

Modal analysis is also a basic concept in structural health monitoring.²⁶⁻³⁰ The recent research focuses on better tools for operational modal analysis,³¹⁻³³ on removing environmental conditions, especially the influence of the temperature,^{34,35} and on automated monitoring,^{36,37} statistical methods,³⁸⁻⁴⁰ and the use of artificial intelligence.⁴¹

The research on bridges at the Federal Institute of Material Research and Testing (BAM) was initiated by Rohrmann,²⁷ and many single-span and multi-span bridges have been measured and monitored mainly with respect to damage detection.⁴²

The experience with bridges from health monitoring is used here for the topic of resonances of railway bridges. This contribution focuses on multi-span bridges and their special effects compared to single-span bridges. What are the typical

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resonance frequencies (resonance frequency patterns), and what are the typical mode shapes of multi-span bridges? Are multi-span railway bridges more dangerous or less dangerous than single-span bridges? Can a multi-span bridge be analyzed by looking at only one span?

The structure of the article is as follows.

The first section presents some theory of multi-span structures with different coupling conditions. The next section, with eight subsections, presents eight measurement examples of multi-span structures: beams, floors, four road bridges, and two railway bridges. The examples have different numbers of spans and different structures, such as weakly coupled simply supported spans or parallel beams, continuous spans, and an arch with nearby eigenfrequencies. After the measurement examples, the next section presents the frequency-domain theory for the response to train passages, where characteristic spectra of the train, the transfer functions of the bridge, and the mode shape (modal force) spectra are discussed. This theory is applied to the two railway bridge examples in the next section, where these bridges are analyzed as single-span bridges at first. Then, in the next section, these examples are extended to two-, three-, and multi-span continuous and simply supported bridges, and the differences between single and multiple spans in case of resonant train passages are evaluated.

Theory of Multi-Span Structures

The principal behavior of multi-span floors and bridges can be analyzed by one-dimensional beam models. The differential equation for the displacements u of a beam

$$EI u'''' + \mu u'' = 0 \quad \text{or in frequency domain } \omega$$

$$EI u'''' + \mu \omega^2 u = 0 \quad (1)$$

with bending stiffness EI and mass per unit length μ has four solutions

$$u = A_1 \cos ax + A_2 \sin ax + A_3 \exp(+ax) + A_4 \exp(-ax), \quad (2)$$

if the parameter a is fitted to the differential equation

$$a = \sqrt[4]{EI/\mu\omega^2}. \quad (3)$$

The constants A_i are fitted to the boundary conditions at the supports.

At first, the full coupling of identical beams is considered. The assembly of all beams yields a continuous beam with continuous rotations at the supports. This is demonstrated for 5 m span concrete beams in Fig. 1. For coupled beams, the support conditions depend on the vibrations of the adjacent beams. If two beams vibrate in anti-phase, the mid support is “hinged” (Fig. 1a), whereas the mid support is “clamped” if the beams are in phase (Fig. 1c). Therefore,

the two coupled beams have two eigenfrequencies (Fig. 1e): the hinged–hinged and the higher hinged–clamped eigenfrequency of a single beam. In the case of many coupled beams, more than one eigenfrequency exists. The eigenfrequencies lie in a frequency band between the hinged–hinged and the clamped–clamped eigenfrequency of a single beam (Fig. 1f). Generally, all beams participate in all eigenmodes (Fig. 1b,d).

In contrast to this strong coupling of beams, a weak coupling of simply supported beams is analyzed by inserting rotational springs at the supports. Thus, two adjacent beams can have discontinuous rotations. The resulting spectra and a mode shape are demonstrated in Fig. 1g,h for the same beam model as before. The fourth mode of the 6-span beam clearly shows the additional rotations due to the weaker coupling (Fig. 1g). The different eigenfrequencies of the weakly coupled beams (Fig. 1h) are in a narrower frequency range compared to the strong coupling. Similar multi-span models and their special behavior have also been discussed for the health monitoring of bridges.³⁷

Observations at Eight Example Structures and Bridges

The following eight subsections present modes and eigenfrequencies of eight multi-span structures that have been measured during health monitoring tasks. The purposes of these monitoring tasks and some related results are given as supplementary information. The observations about the mode shapes, eigenfrequencies, and eigenfrequency clusters are used for the train-induced resonances in the second part of the article.

Example 1: Continuous two-span prestressed concrete beam

A two-span prestressed concrete beam with a total length of 24 m (2×12 m) and a width of 1 m has been built on the BAM test area (Fig. 2a). A line of 25 vertical geophones was installed at 1 m intervals along the bridge, and the modal analysis was done by hammer tests. The natural frequencies appear in pairs, where the square roots of the frequencies always keep the same distance (Fig. 2b). The first six modes are presented in Fig. 3 as theoretical modes (lines) and experimental results (square symbols). Each pair of natural frequencies corresponds to the antisymmetric mode with a lower frequency (Fig. 3, left) and the corresponding symmetric mode with a higher frequency (Fig. 3, right). The agreement between theory and experiment is very good.

The prestress of the beam could be varied (max 1200 kN) by the external tendons below the u-shaped cross section (Fig. 4a). The beam was damaged by releasing the prestress, and many cracks developed. A second modal analysis was done in this released state, and then the beam was prestressed again, and a third measurement was done. A clear reduction

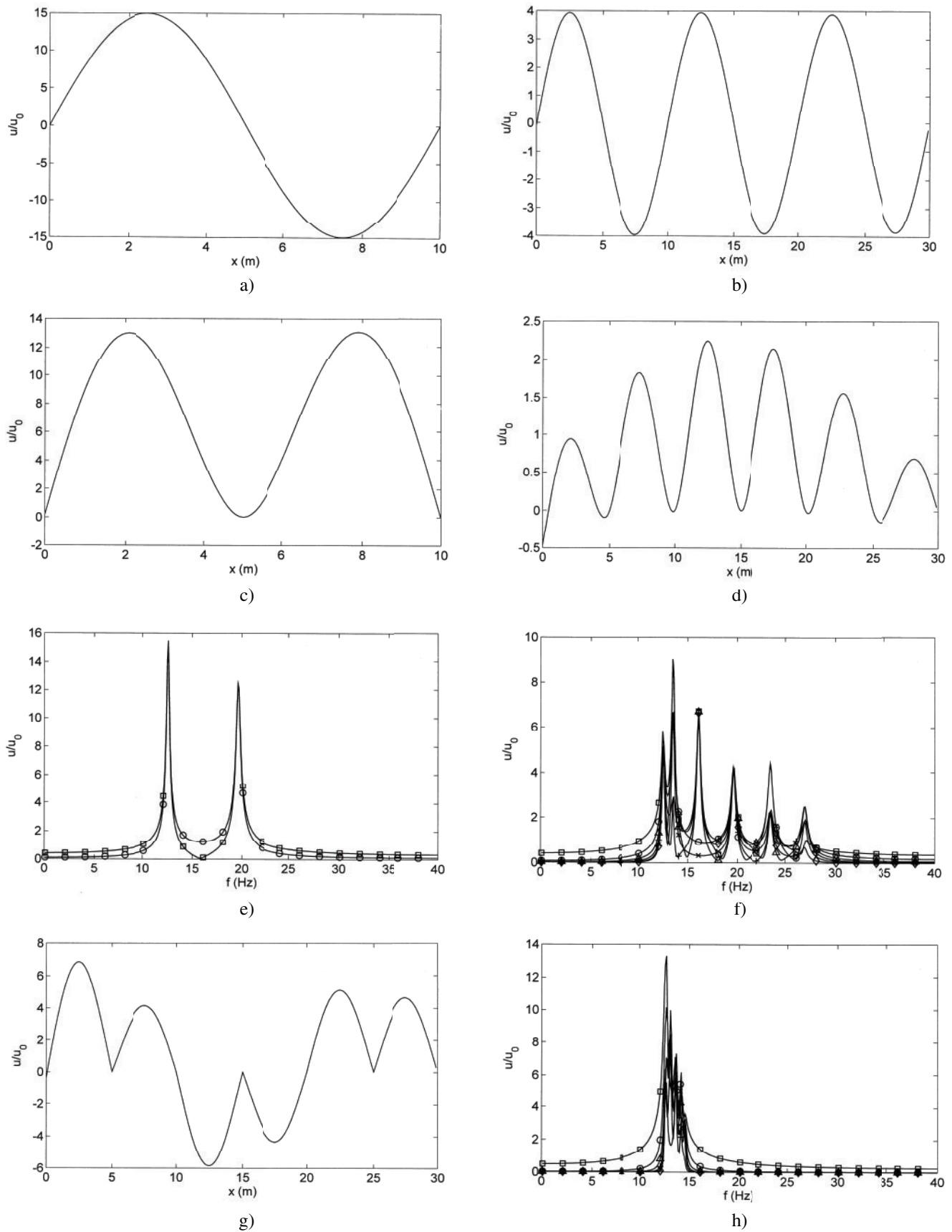
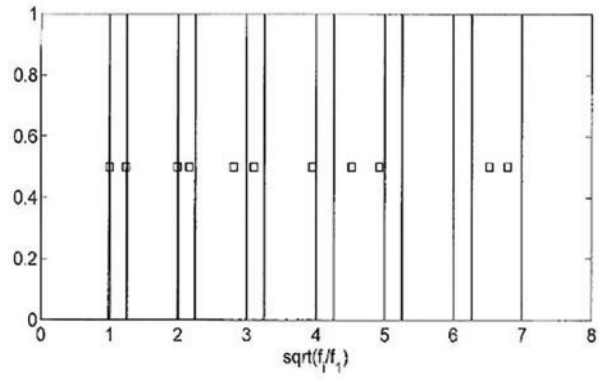


Figure 1. Coupled concrete floor beams of 5-m length, 0.2-m thickness: (a, c, e) 2-span continuous, (b, d, f) 6-span continuous, (g, h) 6-span weakly coupled by rotational springs, (a–d, g) vibration modes, and (e, f, h) spectra



a)



b)

Figure 2. Two-span beam (a) and regular sequence of the natural frequencies (theory) and $\square$ measurements (b)

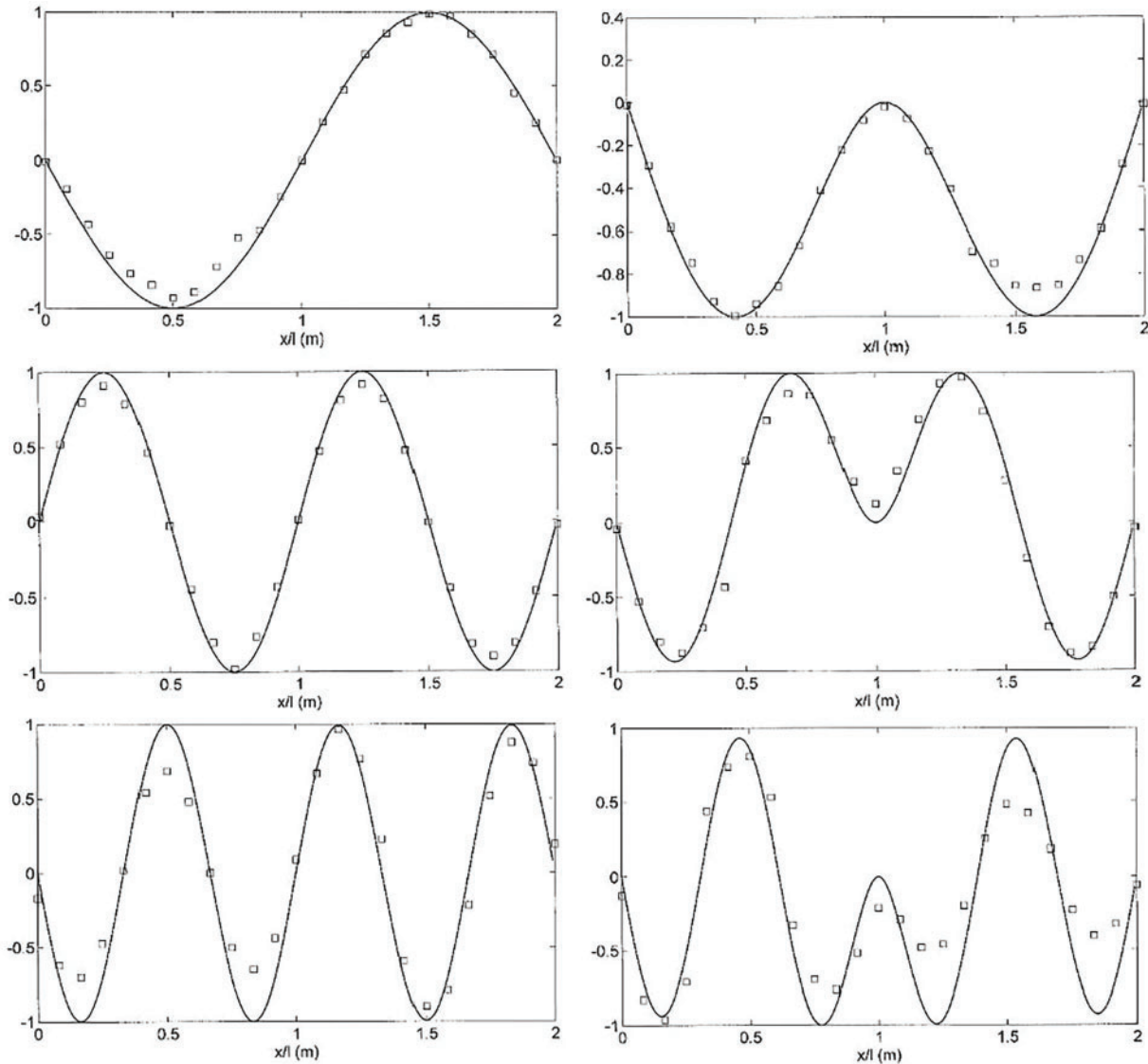


Figure 3. Calculated mode shapes of the two-span beam of length $2l = 24$ m, first, second, and third antisymmetric (left) and symmetric (right) modes and $\square$ measurement results at 3.9, 6.0, 15.4, 18.3, 37.4, 45.9 Hz, with 2.1, 1.3, 1.0, 6.3, 2.2, 4.2% damping

of the first eigenfrequencies was found for the intermediate state without prestress and with subsequent damage, but

the eigenfrequencies are almost identical for the undamaged and damaged states with prestress (Fig. 4b). As the total

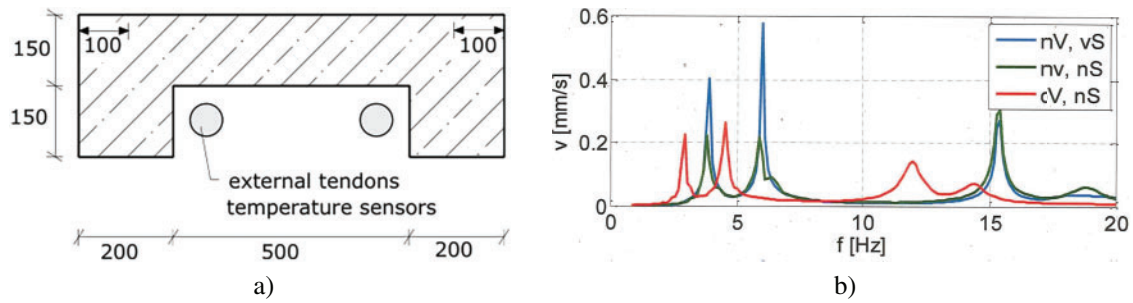


Figure 4. U-shaped cross section of the two-span beam with two external tendons for variable prestressing (a), transfer functions of the beam from hammer impact: with prestress without damage (blue), without prestress after damage (red), and after damage with prestress (green)

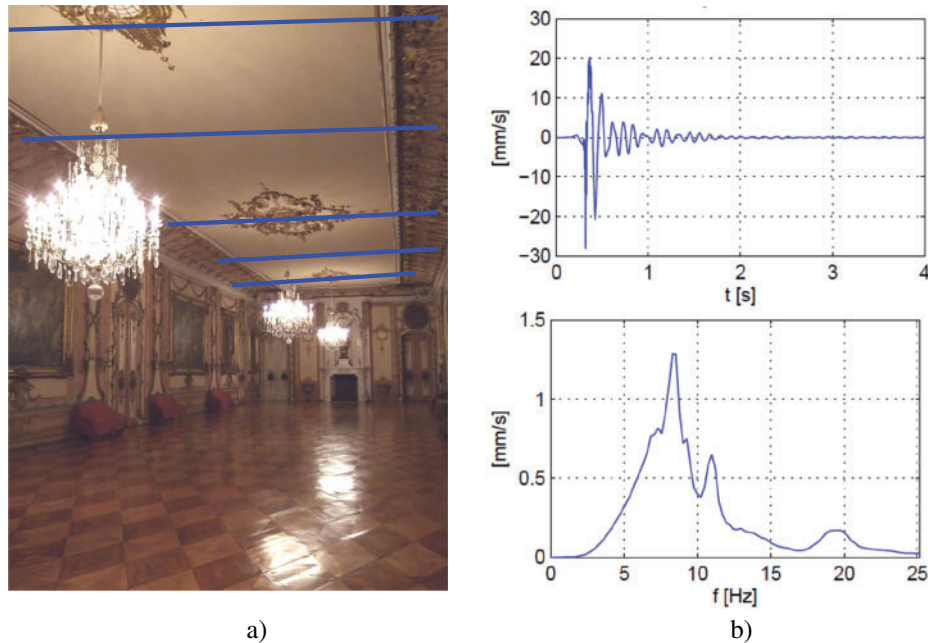


Figure 5. Floor above a large ballroom with 6 rooms on the second story (a) and time history and spectrum of a heel-drop test (b)

loss of prestress gives the maximum crack damage, it can be concluded that cracks under prestress are difficult to detect by modal analysis. The tests were repeated for different lower prestress levels, and the stress levels could be identified.^{43,44}

Example 2: Eleven coupled wooden floor beams (“Neues Palais” in Potsdam)

The large (30 m × 10 m) wooden floor above a ballroom in the “Neues Palais” in Potsdam was dynamically tested (Fig. 5a). Eleven wooden floor beams (0.3 m × 0.4 m) span a length of ten meters and are connected by three layers of floorboards. The floor is divided into six rooms by non-load-bearing walls. The measurement line of 18 sensors along the middle axis of the floor was shifted to three more positions at the quarter lines and along the outer bearing wall. Hammer, heel-drop, and ambient measurements were performed on this floor. In addition to the measurements, a 2-dimensional model of eleven 10-m-long beams was built, which are clamped at both ends and coupled by springs along

the middle axis, and where every second beam is stiffened and loaded by a non-load-bearing wall.

All measured spectra show a maximum region between 5 and 10 Hz with some relative maxima, but without sharp peaks (see Fig. 5b for an example). The wide resonance region indicates the coincidence of several natural frequencies, and a high damping can be assumed for this system. A special method to extract nearby modes was developed and applied to some of the floor modes.⁴⁵

The measured and calculated vibration modes are shown in Fig. 6 as amplitudes along the long axis of the floor. The first mode at about 7 Hz shows a global behavior of the floor, where the floor beams at 2.5, 7.5, ..., 27.5 m have higher amplitudes than the wall beams at 5, 10, ..., 25 m. The second mode at 7.7 Hz consists of vibrations of the floor beams, where every two neighboring rooms are in phase. The next mode at 8.5 Hz consists of vibrations of all floor beams out of phase with their neighbors, while the wall beams are at rest. At a higher frequency of 11.7 Hz, the wall beams show strong amplitudes that are in anti-phase to the floor beams.

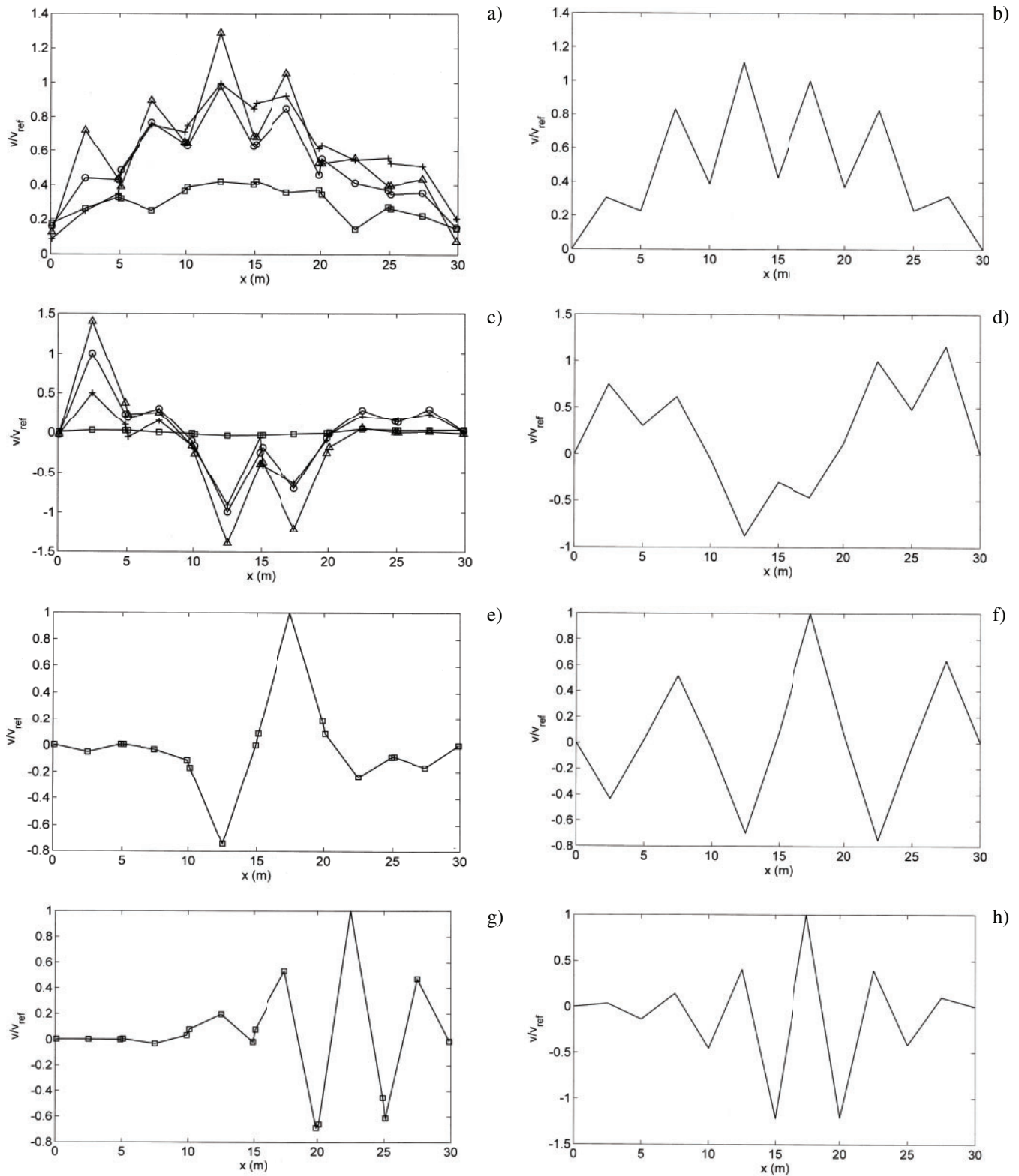


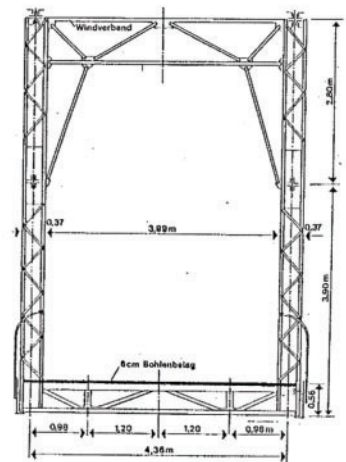
Figure 6. Measured (left) and calculated (right) vibration modes of the 11 coupled floor and wall beams: (a) 6.5 Hz (ambient), (b) 7.2 Hz, (c) 7.7 Hz (ambient), (d) 7.7 Hz, (e) 8.5 Hz (hammer), (f) 8.3 Hz, (g) 11.7 Hz (hammer), (h) 11.7 Hz, Δ middle, $\circ$ outer and $+$ inner quarter axis, and $\square$ outer wall axis in ambient measurements (a, c)

It was found that all floor beams contribute to each natural mode even for a weak coupling of the beams, so that

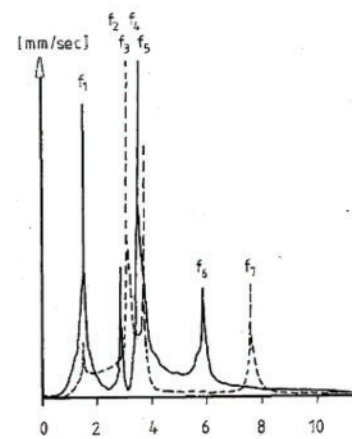
it was impossible to assess the state of each individual beam, neither by global ambient tests nor by local hammer tests.



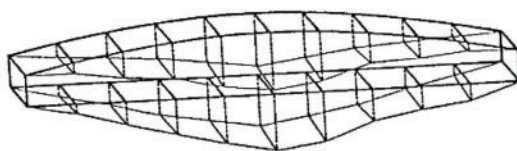
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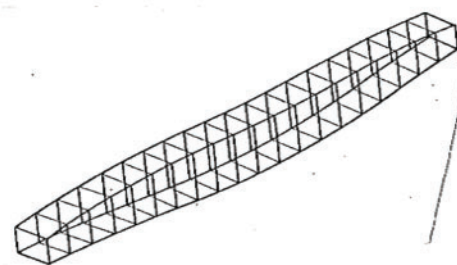
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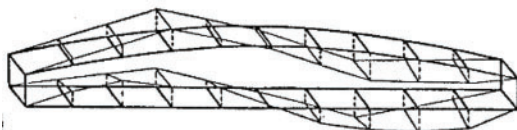
c)



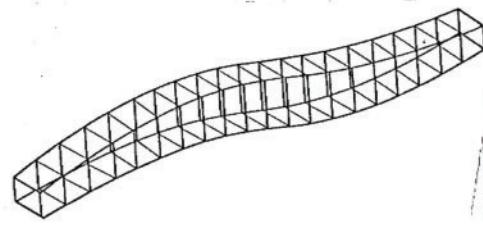
d)



e)



f)



g)

Figure 7. Steel truss arch bridge: view (a), cross section (b), spectra with first eigenfrequencies (c), first vertical eigenmode measured (d) and calculated (e), and second vertical eigenmode measured (f) and calculated (g)

Example 3: Steel truss arch bridge with close eigenfrequencies

The steel truss arch bridge (Fig. 7a,b) is a footbridge with a length of 63.7 m, a width of 4.5 m, and a maximum height of 6.6 m which was sensitive to crowd movement. The positions of dampers should be optimized with the help of the modal analysis. The horizontal and vertical vibrations

due to walking, jumping, wind, and other excitation were measured on the arch and the deck, on both sides of the bridge. The first two vertical modes are shown in Fig. 7d-g. They are the symmetric first bending mode and the anti-symmetric second bending mode. Completely different from a straight beam, the eigenfrequencies of these modes are very close, at 3.16 and 3.63 Hz. This is a ratio of 1.15, whereas

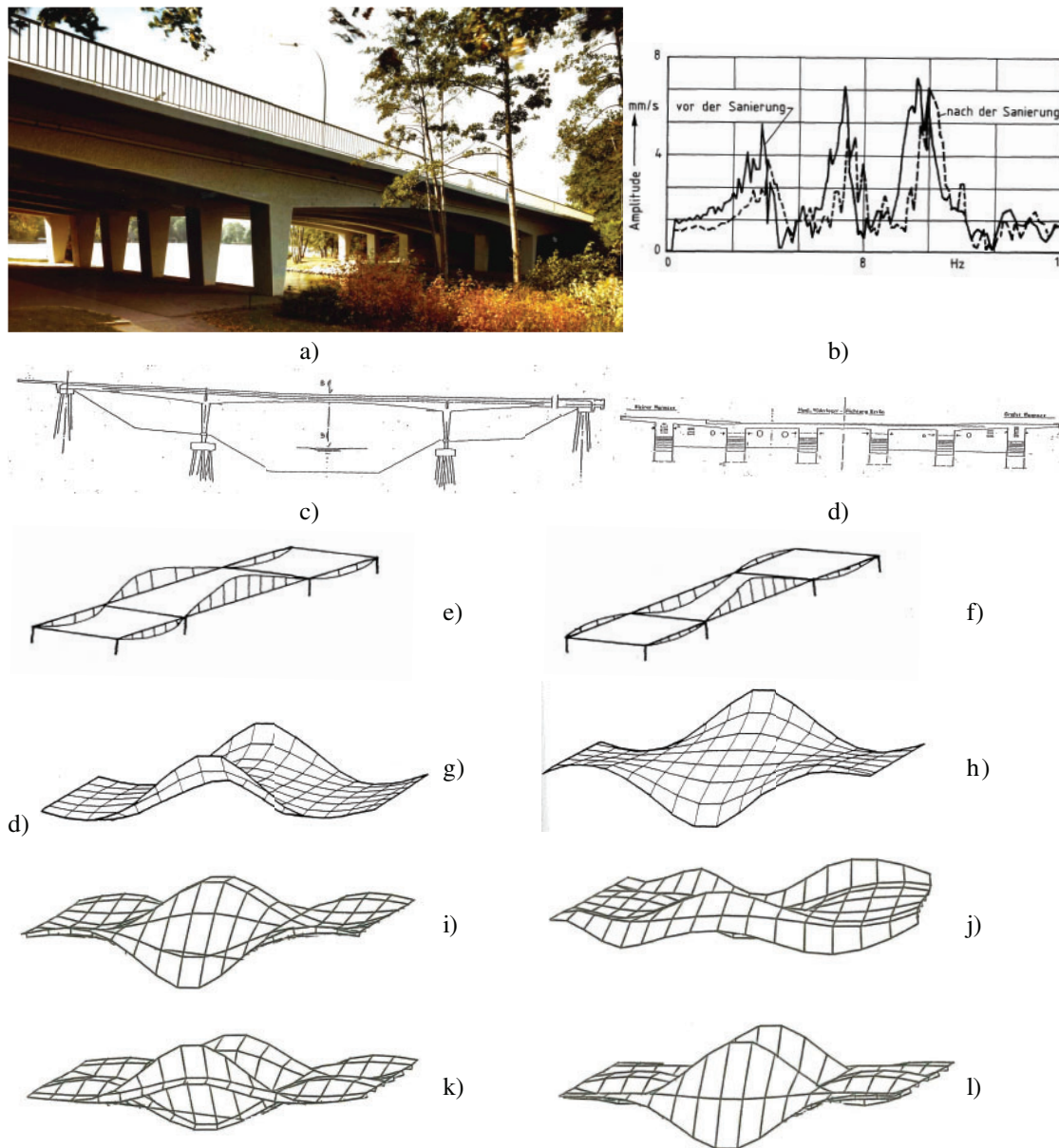


Figure 8. Three-span bridge with six parallel beams: view (a, c), cross section (d), mid-point spectrum before and after renovation (b), the first two mode shapes from measurement (e, 3.9 Hz; f, 4.4 Hz), and the first six mode shapes from finite-element calculation (g–l, 4.1, 4.2, 4.4, 4.5, 4.7, and 4.8 Hz)

the ratio between the second and first eigenfrequency would be 2.75 for a clamped–clamped beam and even 4 for a simply supported beam. The specific behavior of the arch bridge was confirmed by finite-element calculations, which resulted in eigenfrequencies of 2.9 and 3.5 Hz and a ratio of 1.2. Nearby eigenfrequencies of the first and second modes have been found in other measurement examples of arch bridges.^{44–49} Moreover, the symmetric fundamental mode is sometimes missing. This can be explained by the stiffening of the mid-beam due to the curvature, which shifts the first bending mode to a higher frequency. For a strong curvature,

the arch tends to behave like a two-span structure that has a first antimetric mode.

Example 4: Three-span bridge with six parallel beams

The beam-type bridge has a midspan of 35 m and two side spans of 20 m (Fig. 8a). Six prestressed concrete beams are equally distributed across the width of 24 m. The bridge had been repaired by adding steel plates under the beams. The bridge vibrations were measured with 40 geophones on the left and right lanes before and after the repair.

The spectra show a maximum region between 3 and 4 Hz (Fig. 8b) where several modes are assumed. The two first fundamental bending modes could be measured by the two measurement lines. For the first (symmetric or bending) mode (Fig. 8e), the left and right sides of the bridge are in phase, and the shorter side spans are in anti-phase with the midspan. For the second (antimetric or torsional) mode (Fig. 8f), the left and right sides of the bridge are in anti-phase. More modes can be calculated with a 3-dimensional finite-element model (Fig. 8g–i), which consist of different combinations of upward and downward (in-phase and out-of-phase) amplitudes of the six beams. Thus, the full set of fundamental modes could be obtained from the calculation. The amplitudes of the midspan are dominant for all these fundamental modes. The higher resonance frequencies in Fig. 8b belong to higher bending modes of the bridge. An increase in the frequencies was observed after the repair, which indicates a 15% increase in stiffness as a result of the repair.

Example 5: Seven-span continuous box-type highway bridge—health monitoring and modal analysis of the Westend bridge

The Westend highway bridge was built in the early 1960s as a prestressed concrete box-girder bridge. It has a length of 240 m and consists of seven continuous spans of approximately 35 m length and 14 m width (Fig. 9a), which are supported on hinged circular columns, the abutments at the ends, and a clamped column in the middle of the bridge.

The preliminary studies concentrated on the critical third span, where cracks in the box girder and in the shear wall above the column occurred. The modal analysis of this span clearly showed the first and second bending and torsion modes between 2 and 11 Hz (Fig. 9e–h). The first bending mode appears several times with different frequencies. The effects of different excitations, for example, microseismic impacts, sudden release, nearby rail traffic, and light and heavy road traffic, were analyzed, and a first beam model of the whole bridge was updated (Fig. 10).

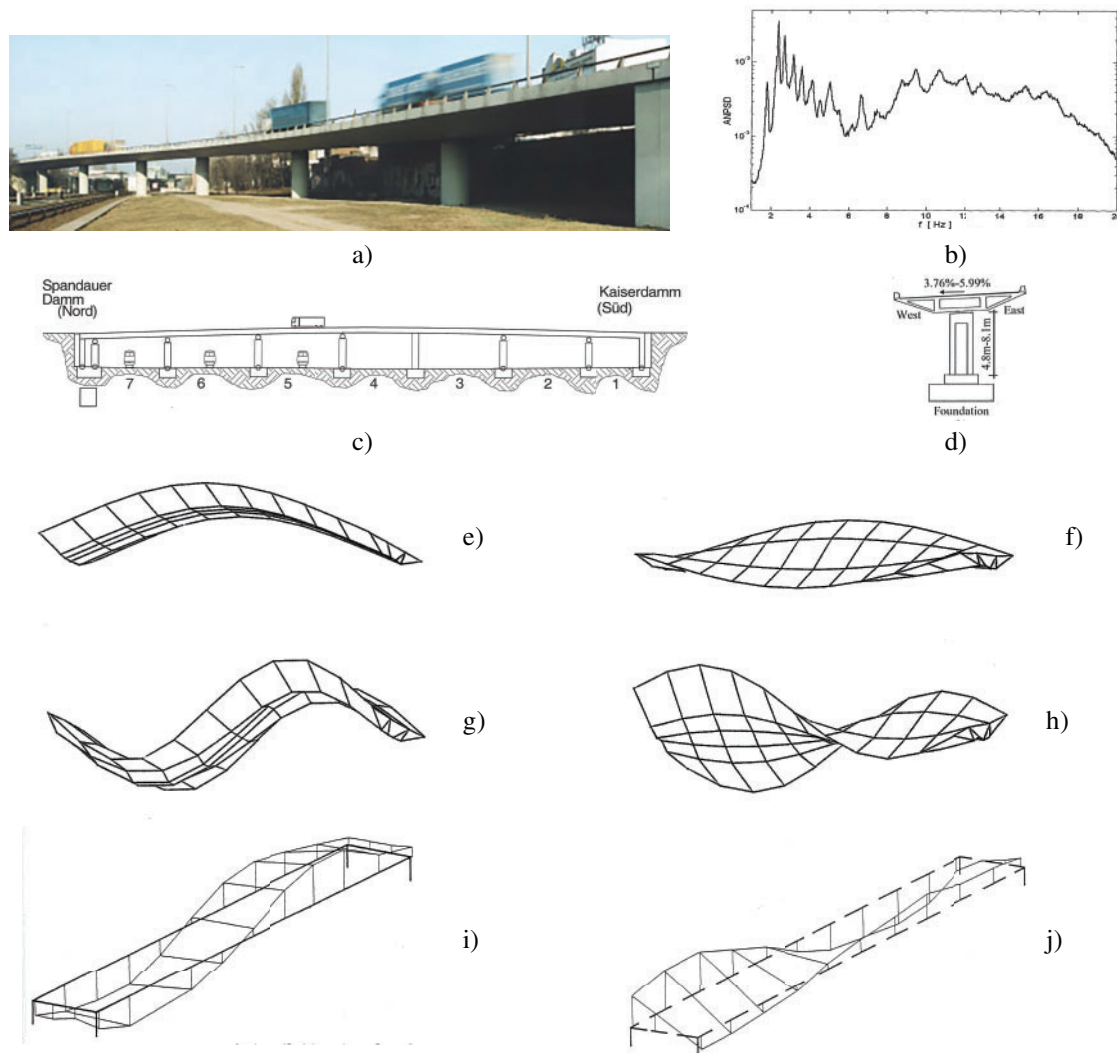


Figure 9. Seven-span highway bridge: view (a,c), cross section (d), averaged spectrum (b), the first four mode shapes of the third span from finite-element calculation (e–h, 2.1, 3.9, 7.1, 10.5 Hz), and the second bending and torsional mode shapes from measurement (i–j, [2.4, 4.6,] 7.6, 11.4 Hz)

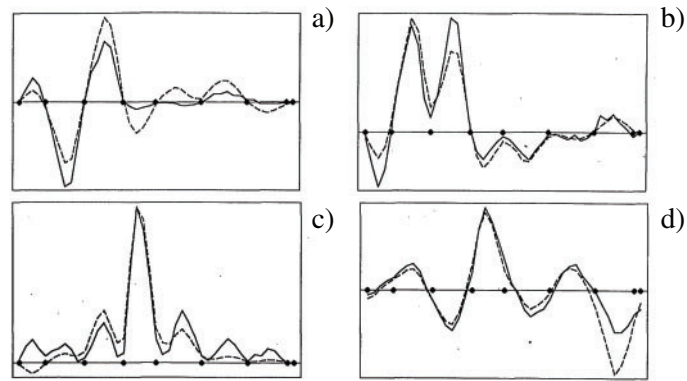


Figure 10. 1D mode shapes of the whole seven-span highway bridge from measurement (continuous line, 2.7, 3.5, 4.6, 6.6 Hz) and calculation

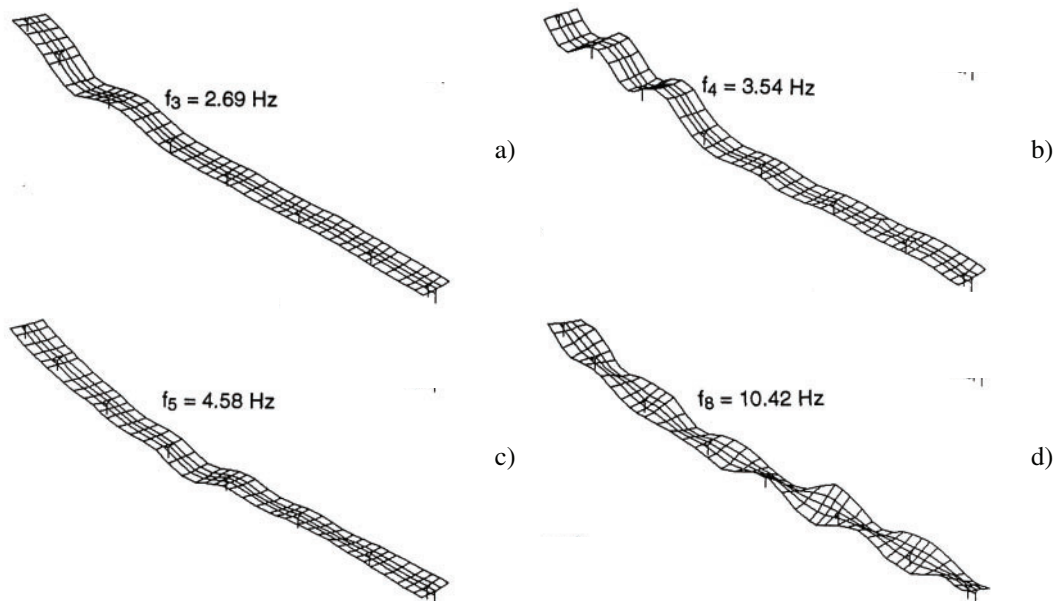


Figure 11. 3D mode shapes of the whole seven-span highway bridge from measurement

A monitoring system was installed and started in 1994. It has been modified and extended several times. It was extended from 16 to 32 and 48 channels, from the critical span 3 to the neighboring spans 1, 2, and 3. Geophones (velocity transducers) are used for the vibrations, eigenfrequencies, and mode shapes. Acceleration sensors and a seismometer were used to check the accuracy at the low-frequency eigenfrequencies. The monitoring software includes the measurement and the immediate evaluation of the raw data.⁵⁰ The characteristics of the vibration, such as max/min values, root-mean-square values, and frequency peaks, are extracted and stored together with selected time segments, single spectra, and averaged spectra. Additional sensors were installed, such as temperature, strain gauges, inclination, and crack-width sensors. A calibration measurement with a 60-t truck has been done, and additional evaluation procedures were implemented for the monitoring of the steadily increasing loads from the road traffic.

Three modal analysis measurement campaigns were performed on the whole seven-span bridge, twice together with

EMPA of Switzerland.⁵¹ The spectrum shows several clearly separated peaks (Fig. 9b). The mode shapes in Fig. 11 show the coupled bending of the different spans, where all spans have alternating minima and maxima at 2.8 Hz, pairs of neighboring spans are in phase at 3.5 Hz, and all spans are in phase at 4.5 Hz. A torsional mode with alternating minima and maxima is found at 10.4 Hz. Note that the three modes at 2.8, 3.5, and 4.5 Hz give almost the same fundamental mode shape at span 3, which often has the highest amplitude of all spans. The different amplitudes of the different spans can be due to the differences in the span lengths of 31 to 38 m, the varying curvature and inclination of the bridge, or the clamped support (rigid pier) between spans 3 and 4, which partly separates the four north spans from the three south spans. Very similar mode shapes were achieved from an updated finite-element model.⁵¹

The stored monitoring data were analyzed for the influence of the traffic loads⁵² and the temperature.⁵³ The monitoring data from 2000 to 2014 were further evaluated by Hu in the context of statistical pattern recognition⁵⁴

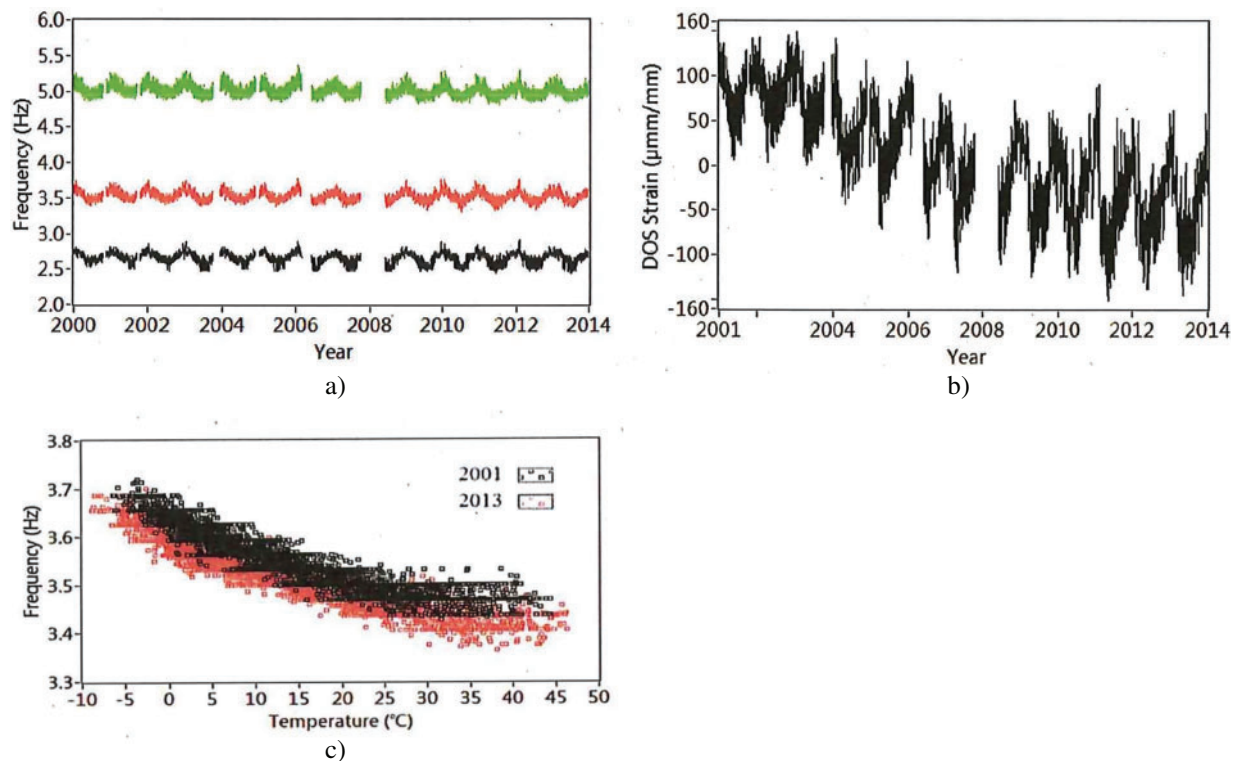


Figure 12. Monitoring data of the Westend bridge: (a) eigenfrequencies, (b) strain of a tendon over 14 years, and (c) influence of the temperature on the eigenfrequency in 2001 (black) and 2013 (red)

(Fig. 12). The influence of temperature was removed, and a trend in some health indices was found, which was related to a loss of prestress. In 2025, the bridge was demolished.

Example 6: Seven-span continuous steel road bridge

The bridge has a length of 264 m and a width of 18 m (Fig. 13a). The main mid-span is 66 m long, and on both sides are three shorter spans of half the length (33 m). There are two steel boxes along the bridge, regularly spaced cross beams, and a concrete deck. A modal analysis was performed after the construction to provide a reference for a later comparison with a possibly damaged state. Five measurement axes with 2×33 geophones were measured to represent the mode shapes. Five examples are shown in Fig. 13c. The first two modes are fundamental bending and torsional modes of the longer mid-span, with small contributions from the nearest side spans. The third mode consists of coupled bending vibrations of both outer spans. The mode at 4.58 Hz is the second torsional mode of the long mid-span and at the same time the first torsional mode of the shorter side spans. The bending mode at 8 Hz has regular minima and maxima at equal distances. The wavelength is half the side span and a quarter of the mid-span, so this mode is the second bending mode for the shorter spans and the fourth bending mode for the long mid-span. At this frequency, the bridge behaves like a bridge with eight equal spans and an additional support at the mid-span.

Example 7: Long three-span concrete rail bridge

The continuous rail bridge (Fig. 14a) has an overall length of 300 m, where the support distances are 135 m for the mid-span and 82 m for the side spans.⁵⁵ The concrete box has a height of 4.5–6 m and supports two railway lines. The monitoring of the bridge during some years includes strains, inclinations, temperatures, acceleration, and 13 geophones along the bridge, which were used to find the vibration modes during ambient excitation and the passages of freight and high-speed trains. The fundamental mode at 1.95 Hz corresponds only to the vibration of the longest mid-span (Fig. 14c). The modes between 3 and 3.5 Hz (Fig. 14d–g) have maxima at the shorter side spans in different combinations, but also some contributions from the mid-span. At 4.85 Hz, the second bending mode of the mid-span was measured. This bridge will be further considered under train passage in the second part of the article.

Example 8: Short two-span concrete rail bridge

The short two-line railway bridge “Arroyo Bracea I” (Fig. 15a) was analyzed by the Spanish colleagues⁵⁶ for the typical resonance effect from high-speed trains. It consists of two simply supported spans of 15.25 m length and 11.6 m width. Each span consists of five concrete beams of 1 m height and a concrete deck of 0.25 m thickness. The modal analysis was done on one of the two spans with 3 axes of 3–4 accelerometers. After stabilization, three modes were found in close neighborhood (Fig. 15b), all of which are first bending modes of the beams. All beams are in phase

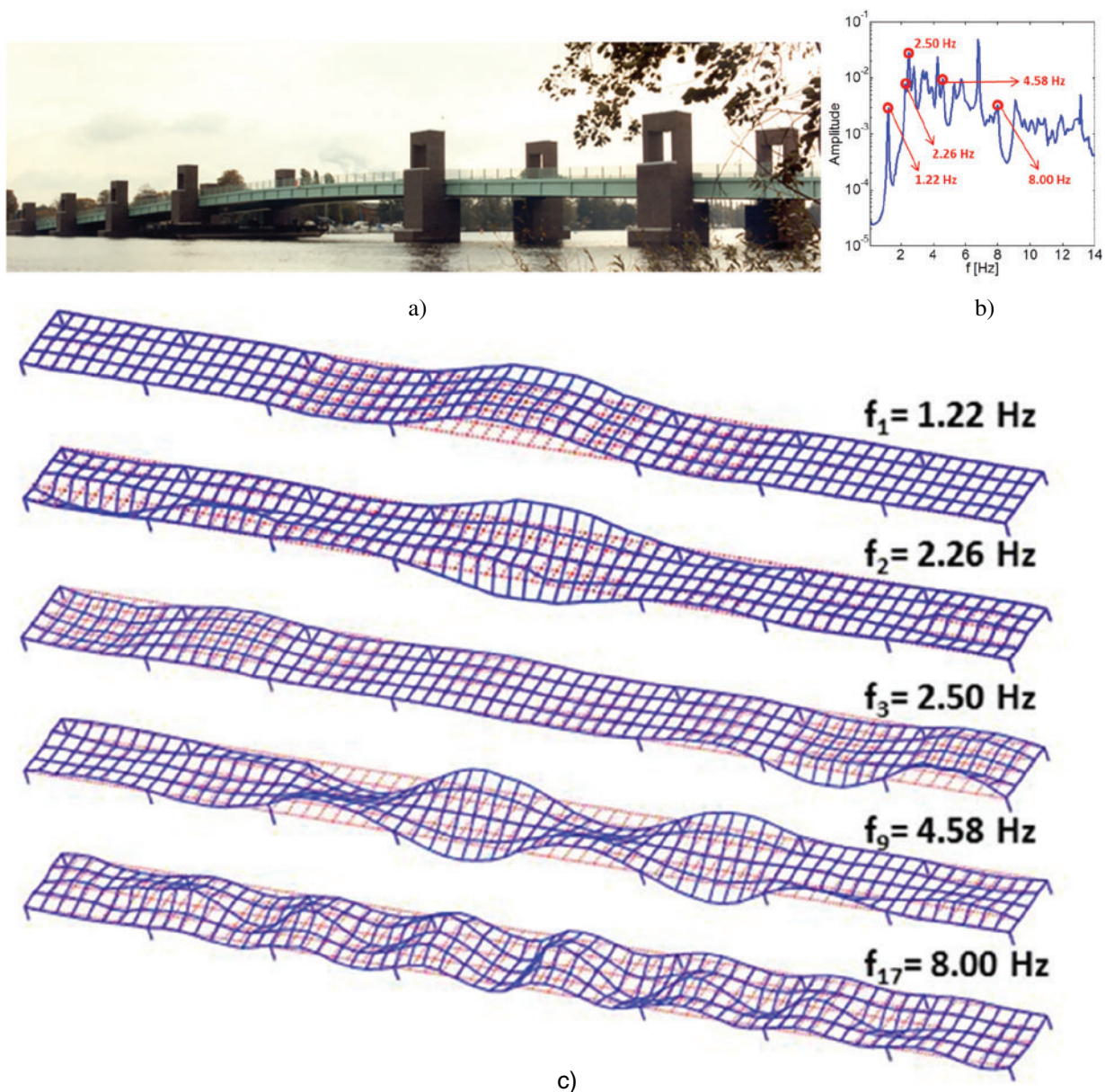


Figure 13. Seven-span regular continuous bridge (a), averaged spectrum (b), and 3D mode shapes from measurement (c)

at 9.25 Hz, the outer beams are out of phase at 10.63 Hz (“torsion”), and the outer beams are in phase while the mid beam is out of phase at 12.75 Hz. The same mode shapes were calculated by two different models of this skew bridge, producing identical or slightly different natural frequencies (see Fig. 10 and Table 4 in Ref⁵⁶).

Summary of observations

The multi-span bridges with n spans generally have n fundamental modes, where the spans contribute with different positive and negative amplitudes.

- In the case of equal, simply supported, weakly coupled spans, a cluster of eigenfrequencies occurs (Example 2).
- The arch bridge (example 3) has nearby frequencies of the first and second bending modes.

- The beam-type bridges (examples 4 and 8) also have a cluster of eigenfrequencies.
- The continuous beams and bridges (examples 1, 5, and 6) show modes with mixed simple and clamped support conditions and more separated fundamental eigenfrequencies.

Theory for the Bridge Resonance from Train Passages

The problem of resonances of single-span or multi-span railway bridges is now addressed. The passage of a train excites the different modes of the railway bridge with (angular) eigenfrequencies ω_j and mode shapes $\omega_j(x)$. The solution for the bridge vibration can be expressed in the frequency

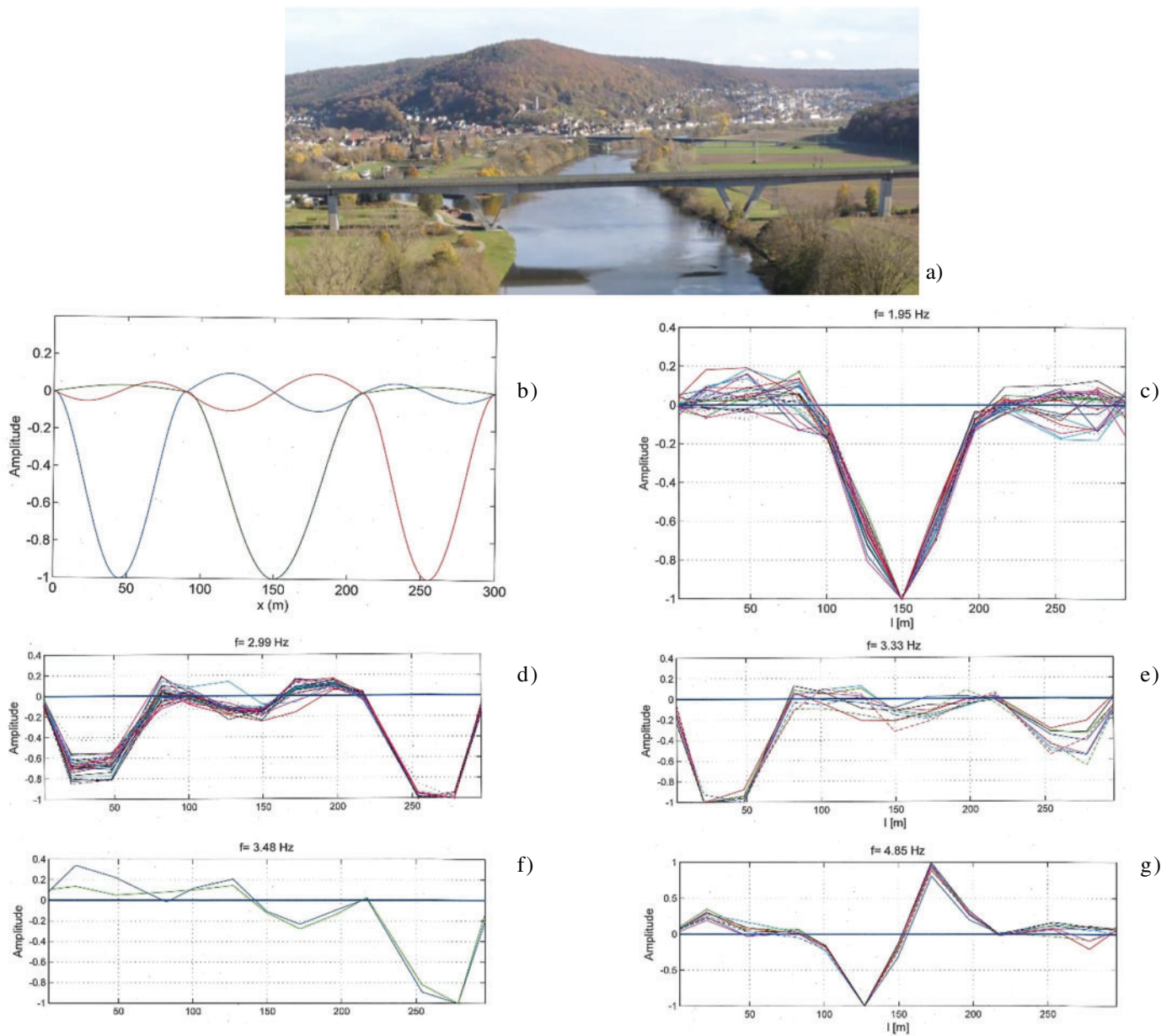


Figure 14. Three-span railway bridge (a), theoretical mode shapes of the first (green) and the second and third (blue and red) eigenfrequencies (b), measured mode shapes of the first eigenfrequency (c, 1.95 Hz), and the next four eigenfrequencies from measurement (d–g, 2.99, 3.33, 3.43, 4.85 Hz)

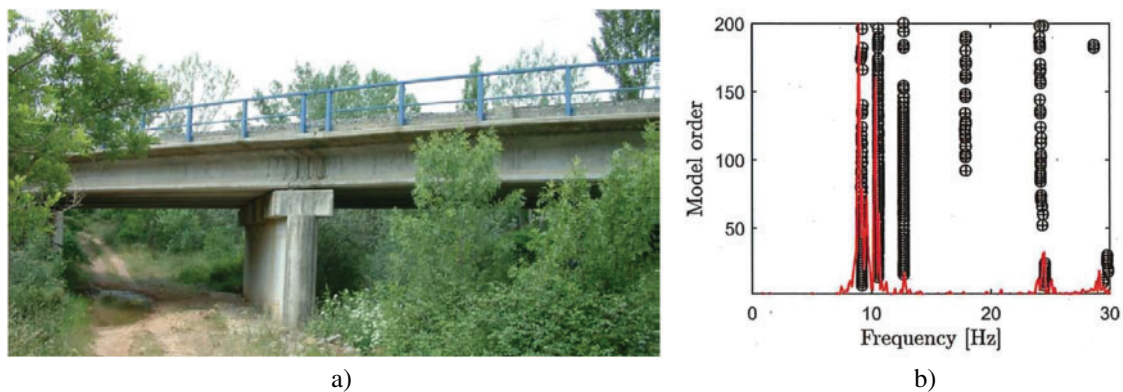


Figure 15. Two-span simply supported bridge (a) and stabilization diagram from stochastic subspace identification (b)

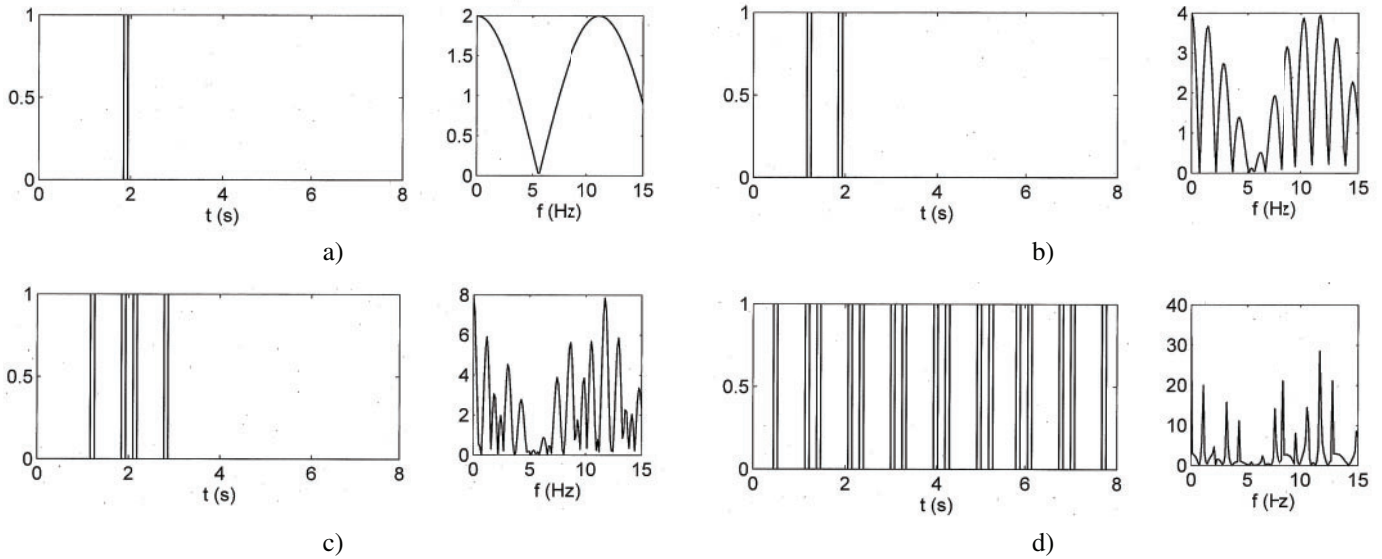


Figure 16. Axle sequence in time and frequency domain for (a) a bogie (axle distance 2.5 m), (b) a car (bogie distance 17 m), (c) two cars (car length 25 m), and (d) eight cars traveling at 100 km/h

domain as the superposition of the eigenmodes, where each contribution is a product of three spectra:

$$v(\omega) = \sum_j v_j(\omega) = F_S X(\omega) \sum_j W_j(\omega) H_j(\omega) \quad (4)$$

where F_S is one static axle load, $X(\omega)$ is the axle-sequence spectrum of the train, $W_j(\omega)$ is the spectrum of the mode shape $\omega_j(x)$, and $H_j(\omega)$ is the corresponding transfer function of the bridge. This result is derived in detail in⁵⁷. It holds for any type of railway bridges: simply supported, elastically supported, integral, single-span, multi-span, and continuous bridges. The time-domain solution is obtained via the inverse Fourier transform.

The axle-sequence spectrum of the train is

$$X(\omega) = \sum_{k=1}^n A_k \exp(-i\omega T_k) \quad (5)$$

where T_k is the delay time of the k th axle and A_k is an amplitude factor compared to the single axle. Fig. 16 shows the axle sequence and the axle-sequence spectrum for a bogie, a car with two bogies, two cars, and for a train with eight cars. The single bogie shows characteristic zeros, the whole car shows many characteristic maxima (and zeros), and these maxima turn into sharp peaks for the whole train.

The mode-shape spectrum is the frequency-dependent part of the modal force spectrum. The modal force in general is

$$F_j(\omega) = \int F'(x, \omega) \omega_j(x) dx \quad (6)$$

and the modal component F_j of the moving constant load F_S is

$$F_j(t) = \int F_S \delta(x - v_T t) \omega_j(x) dx = F_S \omega_j(v_T t) \quad (7)$$

$$F_j(\omega) = F_S W_j(\omega)$$

in time and frequency domain, where v_T is the train speed. The mode-shape spectrum $W_j(\omega)$ is the Fourier transform

of $\omega_j(v_T t)$, the time history of a single unit modal force. The mode-shape spectrum $W_1(\omega)$ of a single bridge span has characteristic zeros due to the bridge length L or the passage time T , respectively. The mode-shape spectrum of a multi-span bridge consists of the sum of the mode-shape spectra of all bridge spans:

$$W(\omega) = \sum_{k=1}^n W_k(\omega) \exp(-i\omega T_k) \quad (8)$$

In the case of identical bridge spans, this is simply

$$W(\omega) = W_1(\omega) \sum_{k=1}^n A_k \exp(-i\omega T_k) = W_1(\omega) Y(\omega) \quad (9)$$

a bridge-sequence spectrum $Y(\omega)$. This is illustrated for two out-of-phase spans and for two in-phase spans in Fig. 17. As for the axle-sequence spectrum, there are characteristic zeros, and the maxima can become sharper for more in-phase or out-of-phase bridge spans.

The frequency response function $H_j(\omega)$ of the bridge is

$$\frac{v_j(\omega)}{F_j(\omega)} = H_j(\omega) = \frac{i\omega}{m_j(\omega_j^2 + 2D_j i\omega_j^2 - \omega^2)} \quad (10)$$

for the velocity, and its maximum at the resonance frequency ω_j is

$$\frac{v_j(\omega_j)}{F_j(\omega_j)} = H_j(\omega_j) = \frac{1}{m_j 2D_j \omega_j} \quad (11)$$

where D_j is the modal damping, and m_j is the modal mass

$$m_j = \int m' \omega_j(x)^2 dx \quad (12)$$

The resonance amplification is inversely proportional to the modal mass, the modal damping, and the (angular) eigenfrequency.

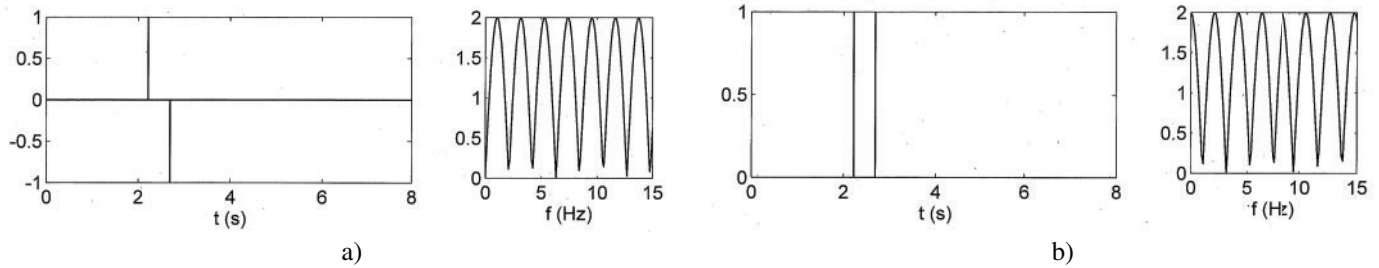


Figure 17. Bridge sequence in time and frequency domain for (a) the out-of-phase mode and (b) the in-phase mode

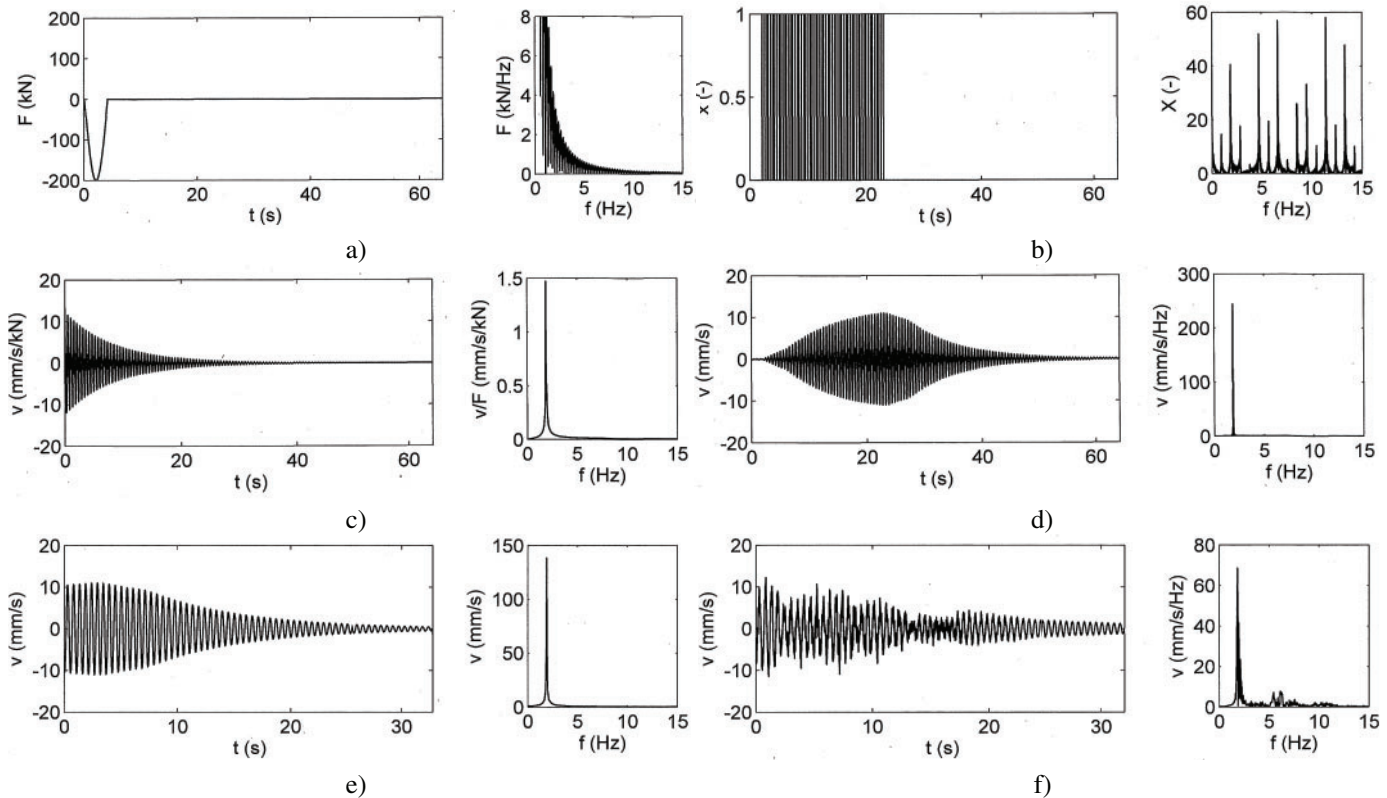


Figure 18. The passage over the long-span railway bridge with $v_T = 100$ km/h, modal force (a), the axle sequence of the twenty 3-axle freight cars (b), impulse and frequency response function of the bridge for $f_l = 1.9$ Hz, $D = 1$ % (c), response of the bridge (d), response of the bridge (partly, e), and measured response (f)

Application to the Rail Bridge Examples

Long bridge with slow freight train

The long rail bridge (example 7; Fig. 14a) has three spans with different lengths and frequencies, and the resonance of the longest mid-span is analyzed as a single-span bridge. The passage of a long freight train at a low speed of 100 km/h over the long bridge lasts about 20 seconds, and a time period of 65.5 s has been investigated. The corresponding modal force (Fig. 18a) shows a passage time of about 2 seconds for a single axle and a spectrum with very narrow minima or zeros. The freight train consists of 20 three-axle cars and has an axle-sequence spectrum (Fig. 18b) with very sharp peaks. If such a peak meets a zero of the modal force spectrum, the cancellation of this resonance would occur. In the present case, the second car-length frequency meets nearly

a maximum of the modal force spectrum, which means an amplification of the resonance. The frequency response function is given for the first bending mode at 1.9 Hz, with a damping of 1 % (Fig. 18c). The mass is at least proportional to the length of the bridge so that the modal mass is high. The resonance of the long-span bridge is reduced by the modal mass, increased by the low resonance frequency, and increased due to the low damping.

The response of the bridge to the passage of the freight train (Fig. 18d) follows from the three spectra in Fig. 6a–c. The spectral density is very high at 240 mm/s/Hz, which is partly due to the very long train. The time history shows an increase during the train passage up to a maximum of 10 mm/s. The following vibration lasts another 20 seconds (Fig. 18e). The resonance with 10 mm/s due to a slow, long

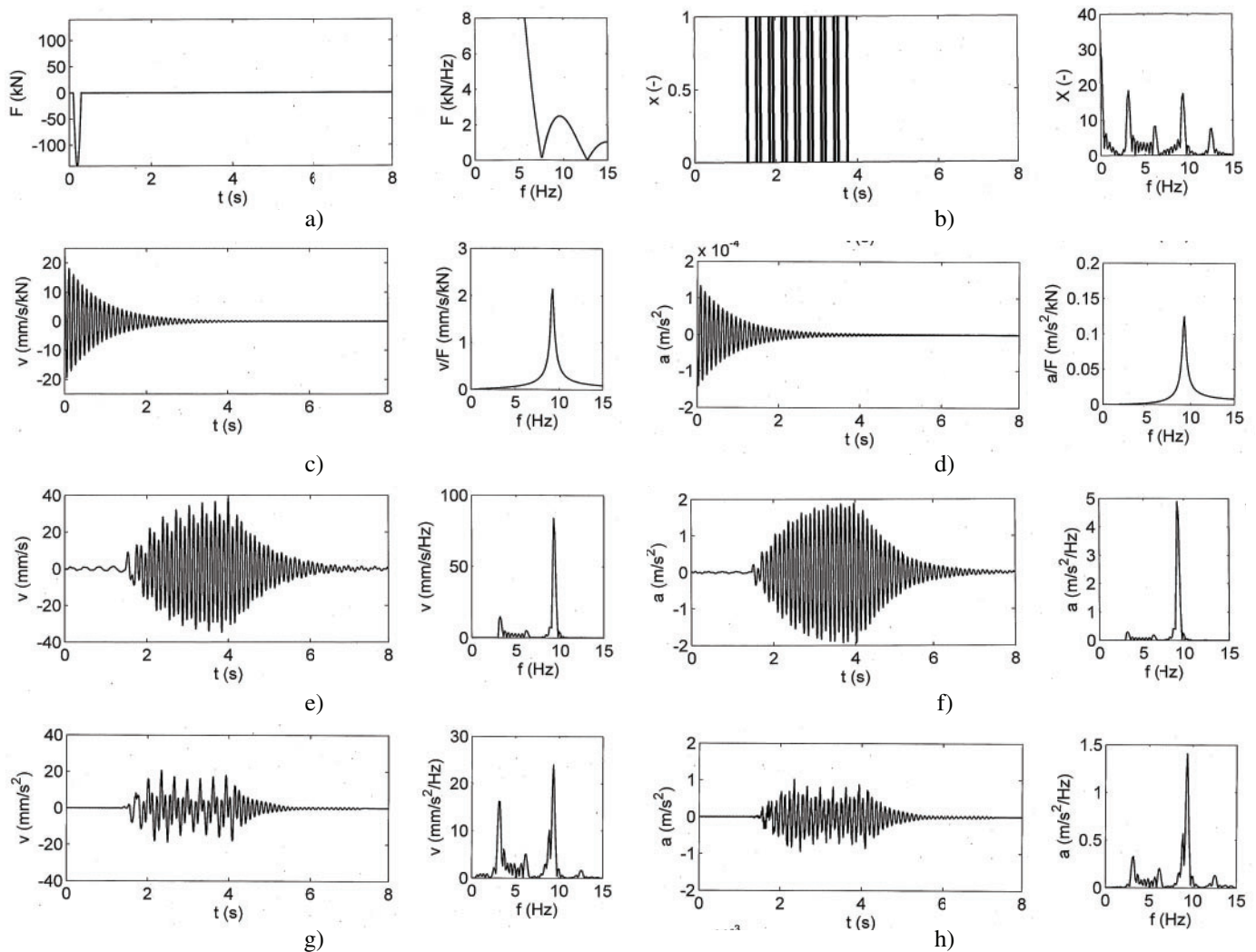


Figure 19. The passage over the short-span simply supported railway bridge with $v_T = 280$ km/h: (a) modal force, (b) the axle sequence of the ICE3 train, (c) velocity response function and (d) acceleration response function of the bridge for $f_l = 9.3$ Hz, $D = 2\%$, (e) velocity response and (f) acceleration response of the bridge, (g) measured velocities, and (h) measured accelerations

freight train on a long-span bridge was also found in the measurements (Fig. 18f).

Short bridge with high-speed train

The Arroyo Bracea bridge (example 8; Fig. 15a) is analyzed for the passage by the Spanish ICE3 train, the S103, at a train speed of 280 km/h. At first, it is analyzed as a single simply supported bridge using the three steps in Fig. 19a–c. The very short passage time yields a high-frequency spectrum of the modal force, where the first side maximum is at about 10 Hz. The axle-sequence spectrum of the ICE3 (Fig. 19b), which is a conventional train with two bogies per car, has a strong third car-length frequency that lies at 9.3 Hz for the train speed of 280 km/h. This is close to the first bending eigenfrequency of the bridge, which is shown in the frequency response function in Fig. 19c for the velocities and in Fig. 19d for the accelerations. The resonance amplification is quite high due to the lower modal mass of the shorter bridge. If all spectra of Figs. 19a–c are put together in

Fig. 19e, the response of the bridge shows high amplitudes of 40 mm/s in the time domain and 80 mm/s/Hz in the frequency domain. This strong resonance builds up during the two seconds of the train passage. The calculated velocity and acceleration responses are compared with the experimental results of the Spanish team⁵⁶ in Fig. 19g and h. The build-up of the resonance is not as strong as in the calculation. The calculated maximum accelerations give higher values of 2 m/s² compared to 1 m/s² for the measurements. The prediction, however, is very similar to the prediction from time-domain calculation in.⁵⁶

The Consequences of Multi-Span Bridges for Rail Traffic

Multi-span bridges of equal or different spans have coupled modes, where other spans contribute more or less to the mode shape of the one dominant span. Therefore, the modal mass of this mode is always higher than the modal mass of

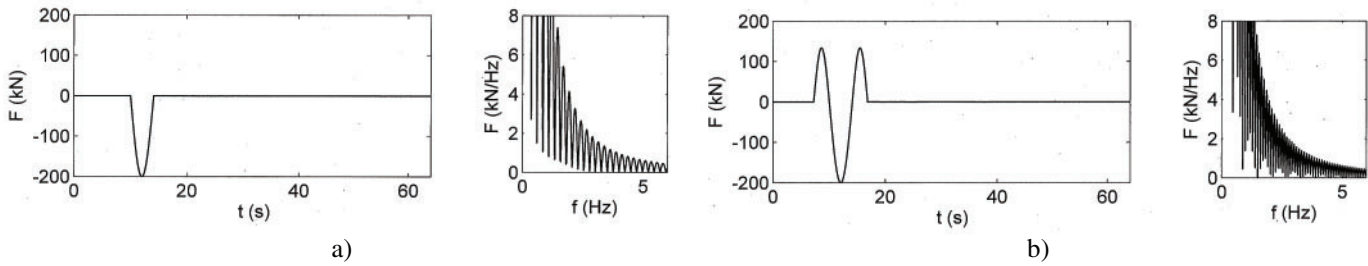


Figure 20. Modal force in time and frequency domain for (a) the long single-span bridge and (b) the long 3-span bridge; 100 km/h train speed

a single-span bridge. In the case of n equal spans, which are all equally present in the mode shape, we have

$$m_{jn} = \int m' w_{jn}(x)^2 dx = n \int m' w_{j1}(x)^2 dx = n m_{j1} \quad (13)$$

where the modal mass is n times the modal mass of the single span. Correspondingly, the maximum amplitude in the frequency response function would be only $1/n$ of the maximum of the single bridge. The modal force, on the other hand, could be higher for the multi-span bridge. The effect on the modal force is analyzed for some examples that are based on the two rail bridges.

Long continuous bridge with three different spans

A 3-span bridge with the dimensions of the long bridge in example 7 (80 m, 120 m, 80 m) is considered. The mode shape of the lowest eigenfrequency (Fig. 20b) is assumed with maxima of 0.67, 1.0, and 0.67 and is compared with the single mid-span mode in Fig. 20a. The corresponding modal force spectra show many maxima due to the length of the bridge and the low speed of 100 km/h. The frequency bands around each maximum are very narrow, especially for the three-span bridge. The envelope of all maxima is similar for the single-span and the three-span bridge. The only difference between the two bridges is the higher modal mass for the three-span bridge, where

$$m_{13} = \int m' w_{13}(x)^2 dx = (1 + 2 \times 0.67 \times 0.67^2) \int m' w_{11}(x)^2 dx = 1.6 m_{11} \quad (14)$$

so that a lower resonance amplitude might be expected for the 3-span bridge. If the contributions of the shorter side spans are smaller, as in the bridge example 6 at 1.2 Hz (Fig. 13), the resonance amplitude is closer to the resonance amplitude of the single-span bridge.

Simply supported bridges with two or more spans

The bridge example from Spain is now idealized as two identical consecutive simply supported bridges. Two fundamental modes are considered here, which have almost the same eigenfrequencies. The corresponding modal force

time histories and spectra are shown in Fig. 21a,b for alternating (out-of-phase) amplitudes and for equal (in-phase) amplitudes. The in-phase mode has a maximum at 10 Hz, which is twice the maximum of a single span (Fig. 19a). The out-of-phase mode has a zero at 10 Hz. The modal mass is twice the modal mass of a single span, so that the effects of the modal mass and the modal force compensate for the in-phase mode, and the resonance is the same as that of a single-span bridge.

If n bridge spans are considered (Fig. 21), n fundamental modes exist. The mode-shape spectra are evaluated at the resonance frequency of the single-span bridge (10 Hz, Fig. 19a). The in-phase modes of the 2-, 3-, and 4-span bridges in Fig. 21b,d,f also have a strong maximum at 10 Hz. The out-of-phase mode of the 3-span bridge (Fig. 21c) has a smaller maximum near 10 Hz, and the out-of-phase modes of the 2- and 4-span bridges (Fig. 21a,e) have a zero at 10 Hz. Besides the zero at 10 Hz, maxima exist above and below that are clearly lower than the maximum for the in-phase mode. Resonances could exist at a lower or higher train speed, but only with a lower resonance amplitude. The same conclusions hold for the mixed mode shapes in Fig. 21g,h. A zero can be found at 10 Hz, and the neighboring maxima are smaller than the maximum of the in-phase mode, which is the most relevant mode with the strongest resonance. The maximum of the in-phase mode-shape spectra (Fig. 21b,d,f) increase proportionally to the number n of spans, so that the effect of the modal mass, which is also proportional to n , is compensated. This means that the resonance of the in-phase mode of the multi-span bridge is the same as the resonance of the single-span bridge. It is expected that these observations can be generalized to all bridges with equal spans.

Multi-span continuous bridges

Similar rules hold for continuous multi-span bridges. Mode shapes that are similar to the mode shapes of the four-span simply supported bridge in Fig. 21e–h have been measured at a four-span continuous highway bridge.⁵⁸ Continuous bridges with more spans can have mode shapes with different amplitudes for different spans (see for example Fig. 1d and Ref^{37,59}). The corresponding effect on the resonance amplitude is analyzed for the most relevant mode, the in-phase mode. The maximum amplitude is 1 and the other

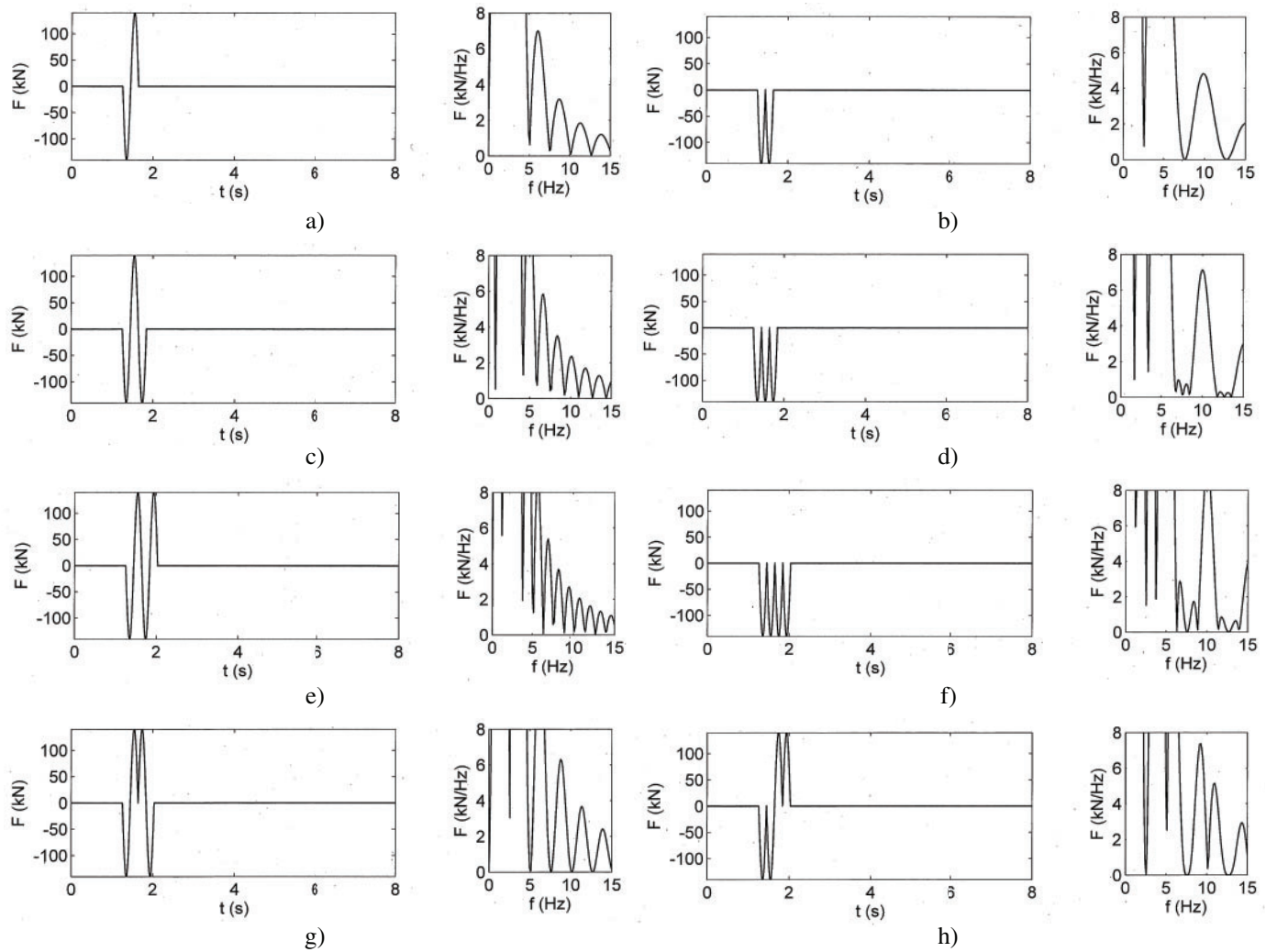


Figure 21. Modal force in time and frequency domain for (a, c, e) the out-of-phase mode of a two-, three-, and four-span bridge, (b, d, f) the in-phase mode of a two-, three-, and four-span bridge, and (g, h) the second and third fundamental modes of the four-span bridge; 280 km/h train speed

amplitudes are $A_k < 1$. The modal mass is

$$\frac{m_n}{m_1} = 1 + \sum A_k^2 < n. \quad (15)$$

and the modal force for the in-phase fundamental mode is

$$\frac{W_n}{W_1} = 1 + \sum A_k < n. \quad (16)$$

The effect on the modal mass is stronger, so that the resonance amplitude is somewhat higher than for a single span.

$$\frac{V_n}{V_1} = \frac{W_n m_1}{W_1 m_n} = \frac{1 + \sum A_k}{1 + \sum A_k^2} > 1. \quad (17)$$

This effect can be studied for the 6-span continuous beam in Fig. 1d. The mid-spans have the highest amplitudes, and the outer spans have the lowest amplitudes. The modal mass is 3.6 instead of 6, and the modal force is 4.5 instead of 6. Finally, the resonance amplification for the 6-span bridge

is 23% higher than for the single-span clamped-clamped bridge. In this case of different amplitudes, the highest fundamental frequency can have a higher resonance amplitude. The ratio between the resonance amplitudes of the multi-span and the single-span bridge depends on the number of spans and the amplitudes A_k . Some calculations have shown that the ratio increases for more spans and smaller A_k . For a specific multi-span bridge, the A_k must be determined from calculated or measured mode shapes. The present rules may help to improve the national and international standard assessment procedures.^{60–63}

For equal amplitudes of all spans, as for the south three-span bridge of example 5 at 3.5 Hz (Fig. 11b) and for the four-span bridge in Ref.⁵⁸ the in-phase mode has similar resonance amplitudes, and all lower fundamental eigenfrequencies have smaller resonance amplitudes compared to the single-span bridge.

Conclusions

Eigenfrequencies and mode shapes of multi-span structures have been analyzed in theory and using eight measurement examples. Whereas a single bridge has only one fundamental mode, multi-span bridges generally have n fundamental modes, where the spans contribute with different positive and negative amplitudes. In the case of equal, simply supported, weakly coupled spans, a cluster of eigenfrequencies occurs. Continuous beams show modes with mixed simply supported and clamped support conditions and separated fundamental eigenfrequencies.

Resonances of railway bridges to train passages have been analyzed in the frequency domain using the axle-sequence spectrum, the transfer function of the bridge, and the modal force (the mode-shape) spectrum. The modal mass increases with the number of spans; the modal force usually does not. Only the in-phase mode has a corresponding increase in the modal force and therefore the same resonance amplitude as a single-span bridge. If the in-phase mode has different amplitudes at the different spans, a higher resonance amplitude is possible. Although higher resonance amplitudes are possible for the in-phase mode, all lower fundamental eigenfrequencies have lower resonance amplitudes than the single-span bridge.

Acknowledgments

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References

- [1] Fryba L. *Dynamics of Solids and Structures under Moving Loads*. Groningen: Noordhoff International Publishing; 1972.
- [2] Olsson M. On the fundamental moving load problem. *J Sound Vibr.* 1991;145(2):299–307. doi:10.1016/0022-460x(91)90593-9.
- [3] Yang Y, Yau J, Hsu J. Vibration of simple beams due to trains moving at high speeds. *Eng Struct.* 1997;19(11):936–944. doi:10.1016/s0141-0296(97)00001-1.
- [4] Savin E. Dynamic amplification factor and response spectrum for the evaluation of beams under successive moving loads. *J Sound Vibr.* 2001;248(2):267–288. doi:10.1006/jsvi.2001.3787.
- [5] Goicolea J, Dominguez J, Gabaldon F, Navarro J. Resonant effects in short span high speed railway bridges: modelling and design issues. *The Proceedings for the 5th International Conference on Structural Dynamics (Eurodyn 2002)*. Lisse: Swets & Zeitlinger. 2002:1077–1082.
- [6] Ju S, Lin T. Resonance characteristics of highspeed trains passing simply supported bridges. *J Sound Vibr.* 2003;267(5):1127–1141. doi:10.1016/s0022-460x(02)01463-3.
- [7] Xia H, Zhang N, Guo W. Analysis of resonance mechanism and conditions of train-bridge system. *J Sound Vibr.* 2006;297(3–5):810–822. doi:10.1016/j.jsv.2006.04.022.
- [8] Yang Y, Lin C, Yau J, Chang D. Mechanism of resonance and cancellation for train-induced vibrations on bridges with elastic bearings. *J Sound Vibr.* 2004;269(1–2):345–360. doi:10.1016/s0022-460x(03)00123-8.
- [9] Museros P, Moliner E, Martinez-Rodrigo M. Free vibrations of simply supported beam bridges under moving load: maximum resonance, cancellation and resonant vertical acceleration. *J Sound Vibr.* 2013;332(2):326–345. doi:10.1016/j.jsv.2012.08.008.
- [10] Xia H, Li H, Guo W, De Roeck G. Vibration resonance and cancellation of simply supported bridges under moving train loads. *J Eng Mech.* 2014;140(5):04014015. doi:10.1061/(asce)em.1943-7889.0000714.
- [11] Lu Y, Mao L, Woodward P. Frequency characteristics of railway bridge response to moving trains with consideration of train mass. *Eng Struct.* 2012;42(5):9–22. doi:10.1016/j.engstruct.2012.04.007.
- [12] Li J, Zhang H. Moving load spectrum for analyzing the extreme response of bridge free vibration. *Shock Vibr.* 2020;9431620(6):1–13. doi:10.1155/2020/9431620.
- [13] Yau J, Martinez-Rodrigo M, Domenech A. An equivalent additional damping approach to assess vehicle-bridge interaction for train-induced vibration of short-span railway bridges. *Eng Struct.* 2019;188:469–479. doi:10.1016/j.engstruct.2019.01.144.
- [14] Stoura C, Dimitrakopoulos E. Additional damping effect on bridges because of vehicle-bridge interaction. *J Sound Vibr.* 2020;476(12):115294. doi:10.1016/j.jsv.2020.115294.
- [15] Xia H, Zhang N, Gao R. Experimental analysis of railway bridge under high-speed trains. *J Sound Vibr.* 2005;282(1–2):517–528. doi:10.1016/j.jsv.2004.04.033.
- [16] Liu K, Reynders E, de Roeck G, Lombaert G. Experimental and numerical analysis of a composite bridge for high-speed trains. *J Sound Vibr.* 2009;320(1–2):201–220. doi:10.1016/j.jsv.2008.07.010.
- [17] Zhang N, Tian Y, Xia H. A train-bridge dynamic interaction analysis method and its experimental validation. *Engineering.* 2016;2(4):528–536. doi:10.1016/j.eng.2016.04.012.
- [18] Gou H, Zhou W, Bao Y, Li X, Pu Q. Experimental study on dynamic effects of a long-span railway continuous beam bridge. *Appl Sci.* 2018;8669(5):1–17. doi:10.3390/app8050669.
- [19] Galvín P, Romero A, Moliner E, De Roeck G, Martinez-Rodrigo M. On the dynamic characterisation of railway bridges through experimental testing. *Eng Struct.* 2020;226(1):111261. doi:10.1016/j.engstruct.2020.111261.
- [20] Cheung Y, Au F, Zheng D, Cheng Y. Vibration of multi-span non-uniform bridges under moving vehicles and trains by using modified beam vibration functions. *J Sound Vibr.* 1999;228(3):611–628. doi:10.1006/jsvi.1999.2423.
- [21] Martinez-Castro A, Museros P, Castillo-Linares A. Semi-analytic solution in the time domain for non-uniform multi-span Bernoulli–Euler beams traversed by moving loads. *J Sound Vibr.* 2006;294:278–297. doi:10.1016/j.jsv.2005.11.009.
- [22] Johansson C, Pacoste C, Karoumi R. Closed-form solution for the mode superposition analysis of the vibration in multi-span beam bridges caused by concentrated moving loads. *Comput Struct.* 2013;119(2):85–94. doi:10.1016/j.compstruc.2013.01.003.
- [23] Martínez-Rodrigo M, Andersson A, Pacoste C, Karoumi R. Resonance and cancellation phenomena in two-span continuous beams and its application to railway bridges. *Eng Struct.* 2020;222(1):111103. doi:10.1016/j.engstruct.2020.111103.
- [24] Tarque N, Goicolea J, Cuevas R, Martinez J. Finite element model updating for the dynamic response of continuous deck railway viaducts leading to efficient digital twins.

- J Phys: Conf Ser (Eurodyn 2023)*. 2024;2647(10):102009. doi:10.1088/1742-6596/2647/10/102009.
- [25] Romero A, Moliner E, Martínez-Rodrigo M, Galvín P. Analytical solution for dynamic response of multi-span bridges under moving loads: an efficient method based on time-discrete analysis using sinc interpolation. *Appl Math Model*. 2026;151A:116491.
- [26] Natke H. *Einführung in Theorie und Praxis der Zeitreihen- und Modalanalyse - Identifikation schwingungsfähiger elastomechanischer Systeme*. Braunschweig: Vieweg; 1983.
- [27] Rohrmann R. Theoretische Ermittlung von Bauwerksschäden mit Hilfe dynamischer Methoden. *Proceedings of Symposium Zerstörungsfreie Prüfung im Bauwesen*. Berlin. 1985:371–382.
- [28] Link M. Updating analytical models by using local and global parameters and relaxed optimisation requirements. *Mech Syst Signal Process*. 1998;12(1):7–22. doi:10.1006/mssp.1997.0131.
- [29] Farrar C, Worden K. An introduction to structural health monitoring. *Philos Trans R Soc A: Math, Phys Eng Sci*. 2007;365(1851):303–315. doi:10.1098/rsta.2006.1928.
- [30] Brownjohn J. Structural health monitoring of civil infrastructure. *Philos Trans R Soc A: Math, Phys Eng Sci*. 2007;365:589–622. doi:10.1142/9789814439022_0001.
- [31] Peeters B, De Roeck G. Reference-based stochastic subspace identification for output-only modal analysis. *Mech Syst Signal Process*. 1999;13(6):855–878. doi:10.1006/mssp.1999.1249.
- [32] Basseville M, Abdelghani M, Benveniste A. Subspace-based fault detection algorithms for vibration monitoring. *Automatica*. 2000;36(1):101–110. doi:10.1016/s0005-1098(99)00093-x.
- [33] Magalhães F. *Operational Modal Analysis for Testing and Monitoring of Bridges and Special Structures*. PhD Thesis, University Porto; 2011.
- [34] Deraemaeker A, Reynders E, De Roeck G. Vibration based structural health monitoring using output-only measurements under changing environment. *Mech Syst Signal Process*. 2007;22(1):34–56. doi:10.1016/j.ymssp.2007.07.004.
- [35] Döhler M, Mevel L, Hille F. Subspace-based damage detection under changes in the ambient excitation statistics. *Mech Syst Signal Process*. 2014;45(1):207–224. doi:10.1016/j.ymssp.2013.10.023.
- [36] Rainieri C, Fabbrocino G. Automated output-only dynamic identification of civil engineering structures. *Mech Syst Signal Process*. 2010;24(3):678–695. doi:10.1016/j.ymssp.2009.10.003.
- [37] Tomassini E, Garcia-Macias E, Reynders E, Ubertini F. Model-assisted clustering for automated operational modal analysis of partially continuous multi-span bridges. *Mech Syst Signal Process*. 2023;200(5):110587. doi:10.1016/j.ymssp.2023.110587.
- [38] Basseville M, Mevel L, Goursat M. Statistical model-based damage detection and localization: subspace-based residuals and damage-to-noise sensitivity ratios. *J Sound Vibr*. 2004;275(3-5):769–794. doi:10.1016/j.jsv.2003.07.016.
- [39] Sohn H, Worden K, Farrar C. Statistical damage classification under changing environmental and operational conditions. *Intell Mater Syst Struct*. 2002;13(9):561–574. doi:10.1106/104538902030904.
- [40] Döhler M, Hille F, Mevel L, Rücker W. Structural health monitoring with statistical methods during progressive damage test of S101 bridge. *Eng Struct*. 2014;69(2082):183–193. doi:10.1016/j.engstruct.2014.03.010.
- [41] Hernández-González I, Ubertini F. Incorporating digital twins and artificial intelligence for next-generation SHM software. In: Rainieri C, Gentile C, Aenlle López M. (Eds.), *Proceedings of the 10th International Operational Modal Analysis Conference (IOMAC 2024)*, *Lecture Notes in Civil Engineering*. Cham: Springer. 2024:515.
- [42] Rohrmann R, Said S, Schmid W, Rücker W, Baeßler M. Zustandsuntersuchung und Dauerüberwachung von Bauwerken mit Hilfe modaler Parameter. In: *VDI Bericht 1550, Experimentelle und rechnerische Modalanalyse sowie Identifikation dynamischer Systeme (VDI Schwingungstagung 2000)*. Kassel; 2000:327–350.
- [43] Viefhues E. *Subspace-Based Damage Detection in Engineering Structures Considering Reference Uncertainties and Temperature Effects*. PhD Thesis, University of Rennes and BAM, Berlin; 2021.
- [44] Liao C, Niederleithinger E, Bernauer F, Igel H, Hadziioannou C. Wave-screening methods for prestress-loss assessment of a large-scale post-tensioned concrete bridge model under outdoor conditions. *Appl Sci*. 2025;15(11):6005. doi:10.3390/app15116005.
- [45] Auersch L, Said S. Methods and phenomena of single and coupled floor vibrations—measurements in apartment and office buildings. *Build Acoust*. 2015;22(2):81–108. doi:10.1260/1351-010x.22.2.81.
- [46] Magalhaes F, Cunha A, Caetano E. Vibration based structural health monitoring of an arch bridge: From automated OMA to damage detection. *Mech Syst Signal Process*. 2012;28:212–228. doi:10.1016/j.ymssp.2011.06.011.
- [47] Rota L, Belleri A, Riva P, Di Prisco M. Operational modal analysis of a RC arch bridge. In: Rainieri C, Gentile C, Aenlle López M. (Eds.), *Proceedings of the 10th International Operational Modal Analysis Conference (IOMAC 2024)*, *Lecture Notes in Civil Engineering*. Cham: Springer; 2024:515.
- [48] Gentile C, Borlenghi P. Dynamic investigations before and after the strengthening of a masonry arch bridge. *Int J Bridge Eng, Manag Res (IJBEMR)*. 2025;2:214250034.
- [49] Rosati I, Reynders E, Lombaert G, Di Ludovico M, Rainieri C. Stiffness contributions to the dynamics of a reinforced concrete arch bridge at low amplitude vibrations. *Structures*. 2026;85(3):111173. doi:10.1016/j.istruc.2026.111173.
- [50] Auersch L, Said S, Rohrmann R. Measurement and evaluation tools for ground and building vibrations from industrial processes, construction work, traffic and other sources. *Proceedings of the 28th International Congress on Sound and Vibration*. Singapore. 2022:198.
- [51] Deger Y, Cantieni R, Pietrzko S, Rücker W, Rohrmann R. Modal analysis of a highway bridge: experiment, finite element analysis and link. *Proceedings of the 13th International Modal Analysis Conference (IMAC)*. Nashville. 1995.
- [52] Rücker W, Rohrmann R, Said S, Schmid W. Load and condition monitoring of a highway bridge in a continuous manner. *Proceedings of the IABSE Symposium Extending the Lifespan of Structures*. San Francisco, California, USA. 1995:1105–1110.
- [53] Rohrmann R, Baeßler M, Said S, Schmid W, Rücker W. Structural causes of temperature affected modal data of civil structures obtained by long time monitoring. *Proceedings of the 18th International Modal Analysis Conference (IMAC)*. San Antonio, USA. 2000.
- [54] Hu W, Tang D, Teng J, Said S, Rohrmann R. Structural health monitoring of a prestressed concrete bridge based on

- statistical pattern recognition of continuous dynamic measurements over 14 years. *Sensors*. 2018;18(12):article-4117. doi:10.3390/s18124117.
- [55] Simon P, Helmrich M, Herrmann R, et al. Maintalbrücke Gemünden: Bauwerksmonitoring und -identifikation aus einem Guss. *Bautechnik*. 2022;99(3):163–172. doi:10.1002/bate.202100102.
- [56] Galvín P, Romero A, Moliner E, Martínez-Rodrigo M. Two FE models to analyse the dynamic response of short span simply supported oblique high-speed railway bridges: comparison and experimental validation. *Eng Struct*. 2018;167(1–2):48–64. doi:10.1016/j.engstruct.2018.03.052.
- [57] Auersch L. Resonances of railway bridges analysed in frequency domain by the modal-force-excitation, bridge-transfer and axle-sequence spectra. *Eng Structures*. 2021;249(2):113282. doi:10.1016/j.engstruct.2021.113282.
- [58] Avramova A, Gentile C. Long-term operational modal analysis of a steel-concrete composite bridge. In: Döhler M, Melot A, Aenlle López M. (Eds.), *Proceedings of the 11th International Operational Modal Analysis Conference (IOMAC 2025)*; 2025; Gijón: IGOMA.
- [59] Busatta F, Moyo P. Operational modal analysis of periodic structures with application to the Olifants River Viaduct. In: Rainieri C, Gentile C, Aenlle López M. (Eds.), *Proceedings of the 10th International Operational Modal Analysis Conference (IOMAC 2024)*. *Lecture Notes in Civil Engineering*. Cham: Springer. 2024:515.
- [60] UIC Code 776-2. Design requirements for rail-bridges based on interaction phenomena between train, track and bridge; 2009.
- [61] EN 1991-2-2023. Eurocode 1: Actions on structures, traffic loads on bridges and other civil engineering works; 2023.
- [62] RiL 804–9060. Eisenbahnbrücken und sonstige Ingenieurbauwerke planen, bauen und instand halten; 2013.
- [63] IAPF-07. *Instrucción sobre las acciones a considerar en el proyecto de puentes de ferrocarril*. Madrid: Spanish Ministry of Public Works and Transport; 2007.